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A COMPUTERIZED METHOD FOR CALCULATING
FLUTTER CHARACTERISTICS OF A SYSTEM
CHARACTERIZED BY TWO DEGREES OF FREEDOM

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SUMMARY

The formulation given in this paper was developed to calculate flutter frequency and flutter speed for a problem with two degrees of freedom. Two different methods of evaluating the flutter determinant were used so that the results from each method could be compared. Although the method was developed for a particular problem application, it is sufficiently general to solve any flutter system that can be characterized by two degrees of freedom.

INTRODUCTION

The solution method (hereinafter referred to as Program A4868) was developed for computing flutter characteristics of a Pylon (strut) that was installed in the NASA Langley Research Center VSTOL tunnel. This paper presents the development of two methods used to calculate flutter frequency and flutter speed for a problem with two degrees of freedom. Also given in the paper are a program flow diagram, partial program listing, and a sample problem with input and output for the two different methods. A comparison of solution results between the two methods is also given. The flutter equations are derived and provided in the Appendix.

SYMBOLS LIST

a	Distance between elastic axis and center of mass, m (in.)
b	Distance between elastic axis and center of pressure, m (in.)
c	Local chord, m (in.)
$\hat{c}$	Local chord in streamwise direction, m (in.)
C_L	Aerodynamic lift coefficient
C_{L_α}	Aerodynamic lift coefficient slope, 1/rad.
C_m	Aerodynamic pitching moment
EI	Bending stiffness, N - m^2 (lb - in^2)
GJ	Torsional stiffness, N - m^2 (lb - in^2)
g	Structural damping
$\tilde{I}$	Product of distributed mass moment of inertia about center of mass axis and airfoil span, N - sec^2 - m^2 (lb - sec^2 - in^2)
$1/K$	Dimensionless parameter, $\frac{V}{\omega}$
L	Aerodynamic loading on differential segment, newton (lb.)
ℓ	Airfoil span, m (in.) (see sketch)
M	Aerodynamic moment on differential segment, joule (in - lb)
$\bar{m}$	Distributed mass, $\left(\frac{\text{lb - sec.}^2}{\text{in.}}\right)$
S	Reference area, m^2 (in.^{2})
V	Flow velocity, m/sec (in./sec.)
V_{DIV}	Divergence speed, m/sec. (in./sec.)

V_F	Flutter speed, m/sec. (in./sec.)
x	Airfoil spanwise coordinate m (in.)
z	Displacement normal to airfoil section, m. (in.)
α	Angle of attack, rad.
γ	Airfoil (strut) sweep angle, rad.
ρ	Density of airstream, N/m^3 ($lb./in^3$)
ϕ	Phase angle, rad.
ω	Circular frequency, rad./sec.
ω_α	Uncoupled torsional natural frequency, rad./sec.
ω_f	Flutter frequency, rad./sec.
ω_y	Uncoupled bending natural frequency, rad./sec.

Matrix Symbols

[D]	2 x 2 aerodynamics matrix associated with displacements
[M]	2 x 2 mass matrix
[K]	2 x 2 stiffness matrix
[R]	2 x 2 aerodynamic matrix
{q}	2 x 1 matrix of the generalized coordinates

FORMULATION AND SOLUTION OF FLUTTER DETERMINANT

Program A4868 uses two methods to solve the flutter determinant. The equations for a two-degree-of-freedom system (see Appendix A) are given by

$$[M]\ \{\ddot{q}_i\} + (1 + i\ g)\ [K]\ \{q_i\} - V^2\ [D]\ \{q_i\} + V\ [R]\ \{\dot{q}_i\} = \{0\} \quad i = 1, 2 \tag{1}$$

Note: The variables and coefficient matrices of equation (1) are defined in Appendix A.

Assuming solutions of the form

$$q_1 = \bar{q}_1\ e^{i\omega t}$$

$$q_2 = \bar{q}_2\ e^{i(\omega t + \phi)}$$

gives

$$[-\ \omega^2\ [M] + (1 + i\ g)\ [K] - V^2\ [D] + i\ \omega\ V\ [R]]\ \{\bar{q}_i\} = \{0\} \quad i = 1, 2 \tag{2}$$

for a nontrivial solution,

$$|-\ \omega\ [M] + (1 + i\ g)\ [K] - V^2\ [D] + i\ \omega\ V\ [R]| = 0 + i\ 0$$

Noting that $K_{12} = K_{21} = D_{11} = D_{21} = 0$, the flutter determinant becomes

$$\left[\begin{array}{c|c} -\ \omega^2\ M_{11} + K_{11} + i\ g\ K_{11} + i\ \omega\ V\ R_{11} & -\ \omega^2\ M_{12} - V^2\ D_{12} + i\ \omega\ V\ R_{12} \\ \hline -\ \omega^2\ M_{21} + i\ \omega\ V\ R_{21} & \begin{array}{c} -\ \omega^2\ M_{22} + K_{22} + i\ g\ K_{22} \\ -\ V^2\ D_{22} + i\ \omega\ V\ R_{22} \end{array} \end{array}\right] = 0 \tag{3}$$

Solution Method 1.

The first method used to solve for the flutter condition involves the solution of the flutter determinant in terms of a complex variable and the parameter $\frac{1}{K}$ for a given Mach No. This solution is developed as follows:

From equation (1) we can write

$$(- \omega^2 M_{11} + K_{11} (1 + i g) + i \omega V R_{11}) q_1 +$$

$$(- \omega^2 M_{12} - V^2 D_{12} + i \omega V R_{12}) q_2 = 0 \qquad (4)$$

Dividing by $\omega^2 M_{11}$ yields:

$$(- 1 + \frac{K_{11}}{\omega^2 M_{11}} (1 + i g) + \frac{i V R_{11}}{\omega M_{11}}) q_1 +$$

$$(- \frac{M_{12}}{M_{11}} - \frac{V^2}{\omega^2} \frac{D_{12}}{M_{11}} + i \frac{V}{\omega} \frac{R_{12}}{M_{11}}) q_2 = 0 \qquad (5)$$

Defining $\frac{1}{K} = \frac{V}{\omega}$ and $\frac{K_{11}}{M_{11}} = \omega_y{}^2$

Equation (5) becomes:

$$(- 1 + (\frac{\omega_y}{\omega})^2 (1 + i g) + i (\frac{1}{K}) \frac{R_{11}}{M_{11}}) q_1 +$$

$$(- \frac{M_{12}}{M_{11}} - (\frac{1}{K})^2 \frac{D_{12}}{M_{11}} + i (\frac{1}{K}) \frac{R_{12}}{M_{11}}) q_2 = 0 \qquad (6)$$

In a similar manner the 2nd equation from (1) can be written as

$$(- \omega^2 M_{21} + i\, \omega\, V\, R_{21})\, q_1 + (- \omega^2 M_{22} + K_{22} + i\, g\, K_{22}$$

$$- V^2 D_{22} + i\, \omega\, V\, R_{22})\, q_2 = 0 \tag{7}$$

Dividing equation (7) by $\omega^2 M_{22}$ and introducing

$$\frac{K_{22}}{M_{22}} = \omega_\alpha^{\,2}, \quad \frac{1}{K} = \frac{V}{\omega}$$

yields

$$\left(- \frac{M_{21}}{M_{22}} + i\, \frac{1}{K}\, \frac{R_{21}}{M_{22}} \right) q_1 + \left(- 1 + \frac{\omega_\alpha^{\,2}}{\omega^2}\, (1 + i\, g) \right.$$

$$\left. - \left(\frac{1}{K}\right)^2 \frac{D_{22}}{M_{22}} + i\, \frac{1}{K}\, \frac{R_{22}}{M_{22}} \right) q_2 = 0 \tag{8}$$

Multiplying the 2nd term of equation (6) by $\left(\frac{\omega_\alpha}{\omega_\alpha}\right)^2$

yields

$$\left(- 1 + \left(\frac{\omega_\alpha}{\omega}\right)^2 \left(\frac{\omega_y}{\omega_\alpha}\right)^2 (1 + i\, g) + i\, \frac{1}{K}\, \frac{R_{11}}{M_{11}} \right) q_1$$

$$+ \left(- \frac{M_{12}}{M_{11}} - \left(\frac{1}{K}\right)^2 \frac{D_{12}}{M_{11}} + i\, \frac{1}{K}\, \frac{R_{12}}{M_{11}} \right) q_2 = 0 \tag{9}$$

Making the substitution $Z = \left(\frac{\omega_\alpha}{\omega}\right)^2 [1 + i\, g]$ in equations (8) and (9), and letting

$$A = \frac{R_{11}}{M_{11}}, \quad B = \frac{M_{12}}{M_{11}}, \quad C = \frac{D_{12}}{M_{11}}, \quad D = \frac{R_{12}}{M_{11}}$$

$$E = \frac{M_{21}}{M_{22}}, \quad F = \frac{R_{21}}{M_{22}}, \quad H = \frac{D_{22}}{M_{22}}, \quad J = \frac{R_{22}}{M_{22}}, \quad \left(\frac{\omega_y}{\omega_\alpha}\right)^2 = \bar{\omega}^2$$

the flutter determinant becomes:

$$\begin{bmatrix} ((Z\,\bar{\omega}^2 - 1) + i\,\frac{A}{K}) & (-B - (\frac{1}{K})^2\,C) + i\,\frac{D}{K}) \\ (-E + i\,\frac{F}{K}) & (Z - 1 - (\frac{1}{K})^2\,H + i\,\frac{J}{K}) \end{bmatrix} = 0 \qquad (10)$$

Expanding the flutter determinant yields a complex polynomial Z which can be solved using the quadratic formula:

$$\bar{\omega}^2\,Z^2 - ((\bar{\omega}^2\,(1 - H\,(\frac{1}{K})^2) - 1) + (A\,(\frac{1}{K}) + \bar{\omega}^2\,J\,(\frac{1}{K}))\,i)\,Z$$

$$+ (1 + H\,(\frac{1}{K})^2 - A\,J\,(\frac{1}{K})^2 - E\,B - C\,E\,(\frac{1}{K})^2 + D\,F\,(\frac{1}{K})^2) +$$

$$(\frac{1}{K}\,(-A - A\,H\,(\frac{1}{K})^2 - J + E\,D + B\,F + C\,F\,(\frac{1}{K})^2)\,i \qquad (11)$$

Let $X = \frac{1}{K}$ and

$A\,A = \bar{\omega}^2$

$B\,B = (\bar{\omega}^2\,(1 - H\,X^2) - 1) + (A\,X + \bar{\omega}^2\,J\,X)\,i$

$C\,C = (1 + H\,X^2 - A\,J\,X^2 - E\,B - C\,E\,X^2 + D\,F\,X^2) +$

$\quad (X\,(-A - A\,H\,X^2 - J + E\,D + B\,F + C\,F\,(X)^2)\,i$

Then the two complex roots are

$$Z\,P = \frac{-\,B\,B + \sqrt{(B\,B)^2 - 4\,A\,A\,C\,C}}{2\,A\,A} \qquad Z\,N = \frac{-\,B\,B - \sqrt{(B\,B)^2 - 4\,A\,A\,C\,C}}{2\,A\,A}$$

The flutter conditions can be determined by computing values of Z for assumed values of 1/K. The root for which the imaginary part of Z changes sign gives the value of 1/K at which flutter is possible. By definition

$\mathrm{Re}\ Z = \left(\frac{\omega_\alpha}{\omega}\right)^2$, and $\mathrm{Im}\ Z = \left(\frac{\omega_\alpha}{\omega}\right)^2 g$.

Therefore: $g = \frac{\mathrm{Im}\ (Z)}{\mathrm{Re}\ (Z)}$

and

$$\omega = \sqrt{\frac{\omega_\alpha}{\mathrm{Re}\ (Z)}}$$

From a plot of g vs. $\left(\frac{\omega_\alpha}{\omega}\right)^2$ with 1/K as a parameter, then the critical condition $\left(\frac{\omega_\alpha}{\omega}\right)_F$ is determined for the actual value of g for the structure. Knowing ω for the corresponding value of 1/K (at which curve intercepts actual g value) then the flutter speed is obtained from the relation

$$V_F = \frac{\omega_F}{K} \tag{12}$$

Solution Method 2.

The second method used to find the flutter speed involves plotting the solutions of the real and imaginary parts of the flutter determinant. Rewriting equation (9) as

$$\left(-1 + \left(\frac{\omega_\alpha}{\omega}\right)^2 \left(\frac{\omega_y}{\omega_\alpha}\right)^2 (1 + i\,g) + i\left(\frac{1}{K}\right)\left(\frac{R_{11}}{M_{11}}\right)\right) q_1 \tag{13}$$

$$+ \left(-\frac{M_{12}}{M_{11}} - \left(\frac{1}{K}\right)^2 \frac{D_{12}}{M_{11}} + i\,\frac{1}{K}\,\frac{R_{12}}{M_{12}}\right) q_2 = 0$$

and equation (8)

$$\left(-\frac{M_{21}}{M_{22}} + i\,\frac{1}{K}\,\frac{R_{21}}{M_{22}}\right) q_1 + \left(-1 + \left(\frac{\omega_\alpha}{\omega}\right)^2 (1 + i\,g)\right.$$

$$\left. - \left(\frac{1}{K}\right)^2 \frac{D_{22}}{M_{22}} + i\left(\frac{1}{K}\right)\frac{R_{22}}{M_{22}}\right) q_2 = 0 \tag{14}$$

Define $X = \left(\frac{\omega_\alpha}{\omega}\right)^2$

Then the flutter determinant may be written as

$$\begin{bmatrix} -1 + X\left(\frac{\omega_y}{\omega_\alpha}\right)^2 (1 + i\,g) + i\left(\frac{1}{K}\right)\frac{R_{11}}{M_{11}} & -\frac{M_{12}}{M_{11}} - \left(\frac{1}{K}\right)^2 \frac{D_{12}}{M_{11}} + i\left(\frac{1}{K}\right)\frac{R_{12}}{M_{11}} \\ -\frac{M_{21}}{M_{22}} + i\,\frac{1}{K}\,\frac{R_{21}}{M_{22}} & -1 + X(1 + i\,g) - \left(\frac{1}{K}\right)^2 \frac{D_{22}}{M_{22}} + i\left(\frac{1}{K}\right)\frac{R_{22}}{M_{22}} \end{bmatrix} \tag{15}$$

Making the substitutions

$$\bar{\omega} = \frac{\omega_y}{\omega_\alpha}\ , \quad A = \frac{R_{11}}{M_{11}}\ , \quad B = \frac{M_{12}}{M_{11}}\ , \quad C = \frac{D_{12}}{M_{11}}\ , \quad D = \frac{R_{12}}{M_{11}}$$

$$E = \frac{M_{21}}{M_{22}}\ , \quad F = \frac{R_{21}}{M_{22}}\ , \quad H = \frac{D_{22}}{M_{22}}\ , \quad J = \frac{R_{22}}{M_{22}}$$

and expanding the determinant yields:

$$\begin{aligned}
&(\bar{\omega}^2\ (1 - g^2) + 2\ \bar{\omega}\ \ g\ i\ )\ X^2 + (-\ 1 - \bar{\omega}^2\ (1 + (\tfrac{1}{K})^2\ H + g\ J\ (\tfrac{1}{K})) + \\
&(-\ g - \bar{\omega}^2\ (g + (\tfrac{1}{K})^2\ H\ g - X\ J) + A\ (\tfrac{1}{K}) + A\ \tfrac{1}{K}\ g)\ i)\ X + \\
&((1 + (\tfrac{1}{K})^2\ H - (\tfrac{1}{K})^2\ A\ J - E\ B - E\ C\ (\tfrac{1}{K})^2 + F\ D\ (\tfrac{1}{K})^2\ ) + \\
&(\tfrac{1}{K}\ (-\ A - A\ H\ (\tfrac{1}{K})^2 - J + E\ D + F\ B + F\ C\ (\tfrac{1}{K})^2\ ))\ i\ ) = 0
\end{aligned} \qquad (16)$$

Separating the real and imaginary parts into two equations yields:

$$\begin{aligned}
&(\bar{\omega}^2\ (1 - g^2))\ X^2 + (-\ 1. - \bar{\omega}^2\ (1 + (\tfrac{1}{K})^2\ H + g\ J\ (\tfrac{1}{K}))\ X \\
&+ ((1 + (\tfrac{1}{K})^2\ H - (\tfrac{1}{K})^2\ A\ J - E\ B - E\ C - E\ C\ (\tfrac{1}{K})^2 + F\ D\ (\tfrac{1}{K})^2\ ) = 0
\end{aligned} \qquad (17)$$

as the equation of the real part, and

$$\begin{aligned}
&(2\ \bar{\omega}\ g)\ X^2 + (-\ g - \bar{\omega}^2\ (g + (\tfrac{1}{K})^2\ H\ g - (\tfrac{1}{K})\ J) + A\ (\tfrac{1}{K}) + A\ (\tfrac{1}{K})\ g)\ X \\
&+ (\tfrac{1}{K})\ (-\ A - A\ H\ (\tfrac{1}{K})^2 - J + E\ D + F\ B + F\ C\ (\tfrac{1}{K})^2\ ) = 0
\end{aligned} \qquad (18)$$

as the equation of the imaginary part.

By solving these equations by the quadratic formula for X for assumed values of 1/K and plotting the square root of these roots against the parameter $\frac{1}{K} = \frac{V}{\omega}$, the frequency ratio (ω_α/ω) can be determined at the intersection of the curves.

Thus the condition for which flutter can occur is obtained from the relation

$$\omega_F = \frac{\omega_\alpha}{\sqrt{X}} \tag{19}$$

$$V_F = \frac{\omega_\alpha}{K\sqrt{X}} \tag{20}$$

Solution for Flutter Mode Shape

The mode shape for flutter is obtained by substituting ω_F and V_F into equation (2), normalizing on either q_1 or q_2 (say $q_2 = 1.0$) and solving for q_1.

SOLUTION FOR DIVERGENCE SPEED

The solution for the divergence speed is obtained by setting $\omega = 0$, $g = 0$ in equation (3) which gives

$$\begin{bmatrix} K_{11} & -V^2 D_{12} \\ 0 & K_{22} - V^2 D_{22} \end{bmatrix} = 0 \qquad (21)$$

Expand (21) and solving for V gives

$$V_{Div} = \sqrt{\frac{K_{22}}{D_{22}}} \qquad (22)$$

which in this case characterizes a torsional divergence problem.

The divergence mode shape is obtained by substituting V_{div} in equation (2) for $\omega = 0$, $g = 0$ and normalizing on say $q_2 = 1.0$ and solving for q_1.

INPUT DESCRIPTION

The "Namelist" method of input is used; see the Fortran Manual of Control Data System 6400/6600 Series for format instructions. The mode of all variables input is R-real.

1. Namelist: $NAM1 including the following:

Fortran name	Description
M	Array defined as: $M = \omega^2 \frac{m}{2} \begin{bmatrix} c^2 & -ac \\ -ac & a^2 + \frac{I}{m} \end{bmatrix}$
DD	Aerodynamic matrix associated with displacements; defined as: $DD = \frac{1}{4} \rho c \ell C_{L_\alpha} \begin{bmatrix} 0 & c \text{ COS } \gamma \\ 0 & b \text{ COS } \gamma \end{bmatrix}$
R	Aerodynamic maxtrix associated with rates, defined $R = \frac{1}{4} \rho c \ell C_{L_\alpha} \begin{bmatrix} c^2 & bc \\ bc & b^2 \end{bmatrix}$

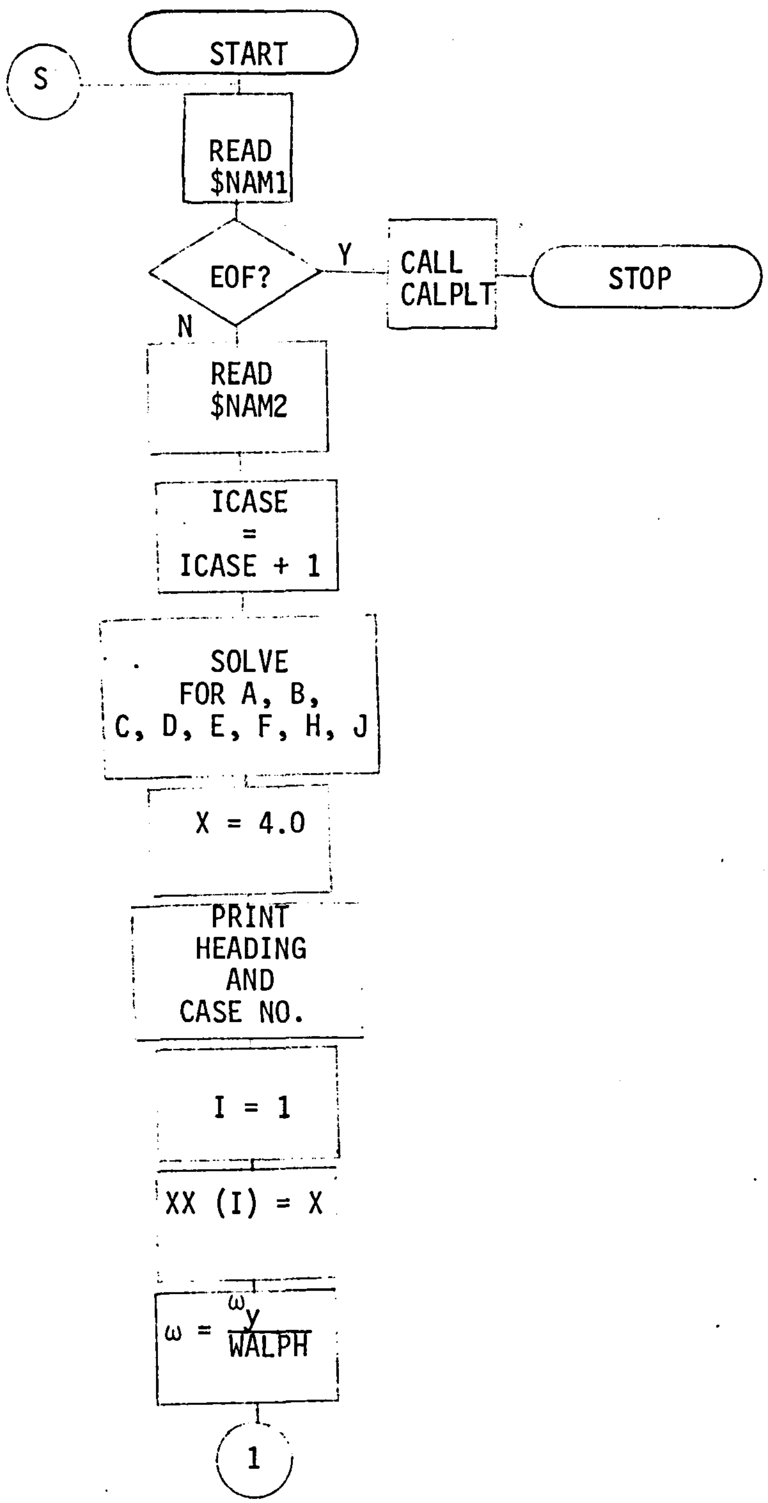
$END NAM1

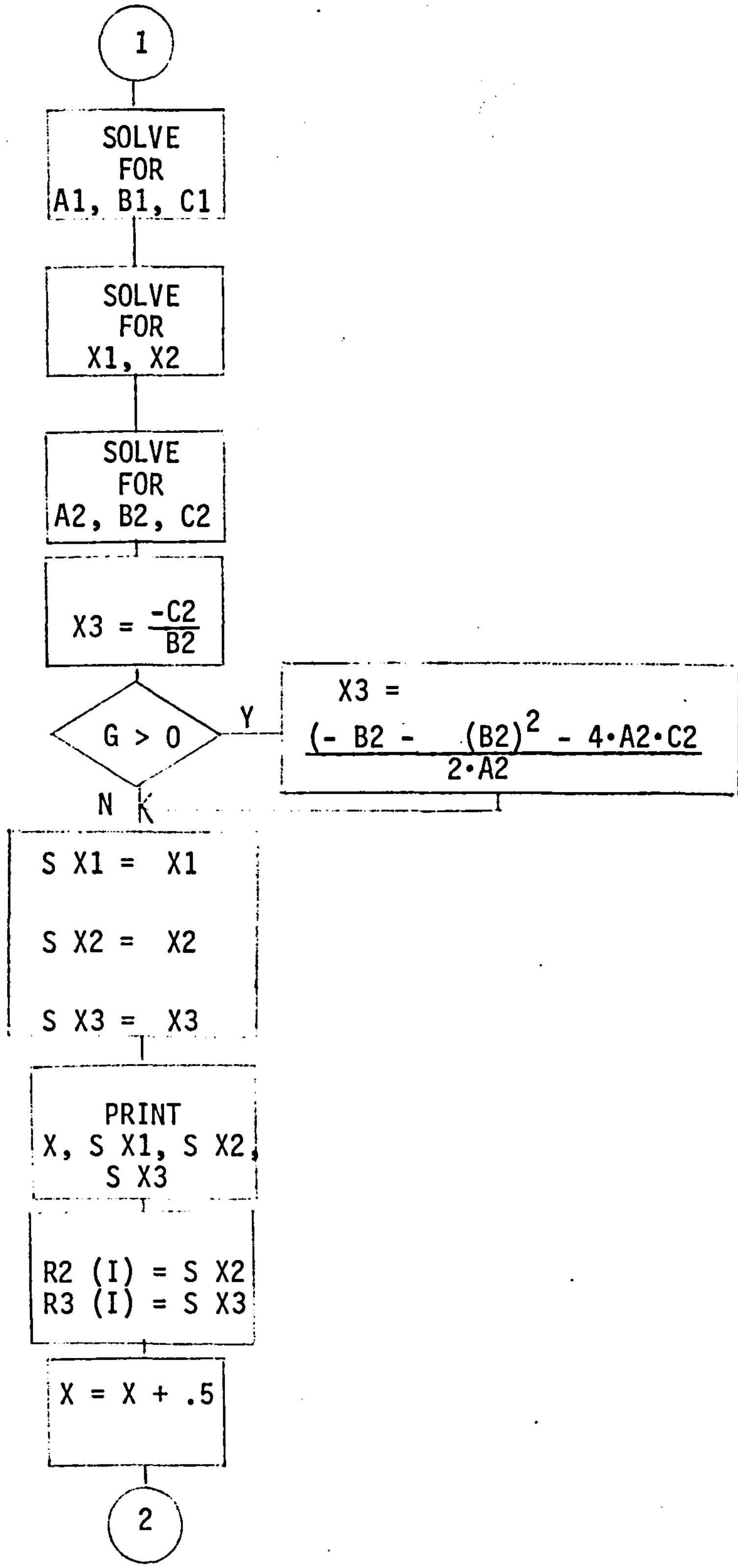
2. Namelist: $NAM2

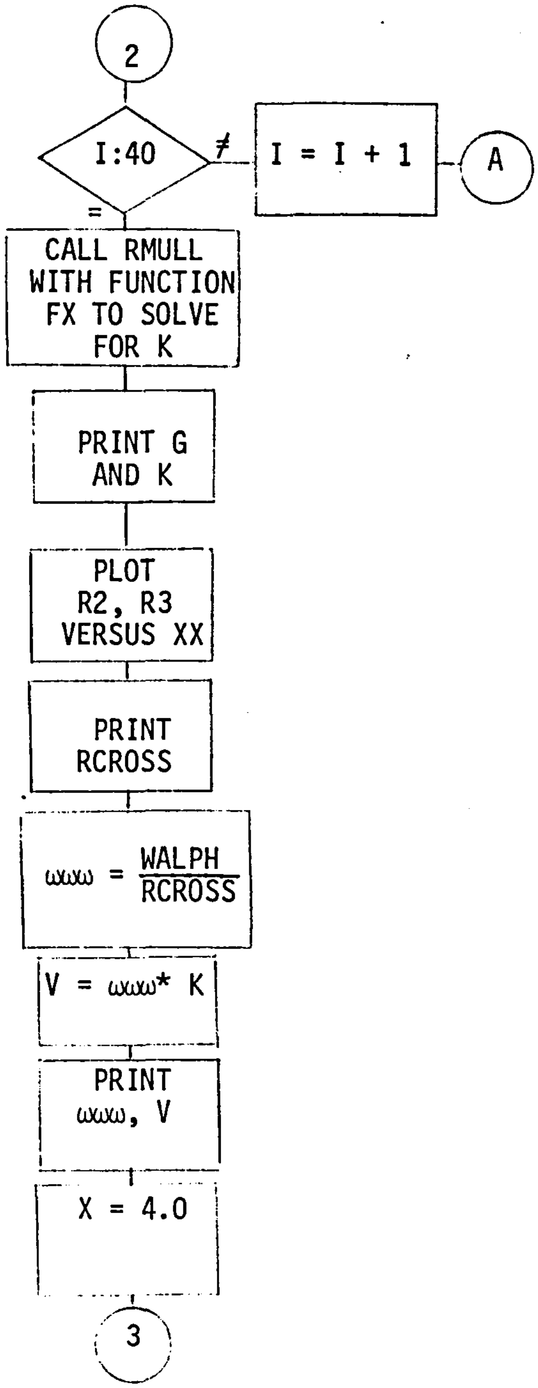
Fortran name	Description
G	Structural Damping, g
WALPH	ω_α = uncoupled natural frequency defined as: $\sqrt{\frac{K\ (2,\ 2)}{M\ (2,\ 2)}}$
Wy	ω_y = uncoupled natural frequency defined as: $\sqrt{\frac{K\ (1,\ 1)}{M\ (1,\ 1)}}$

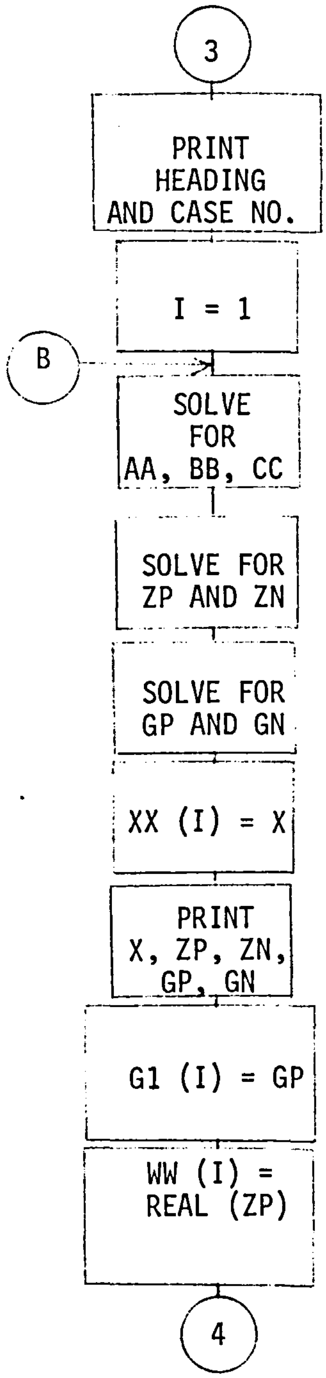
$ END NAM2

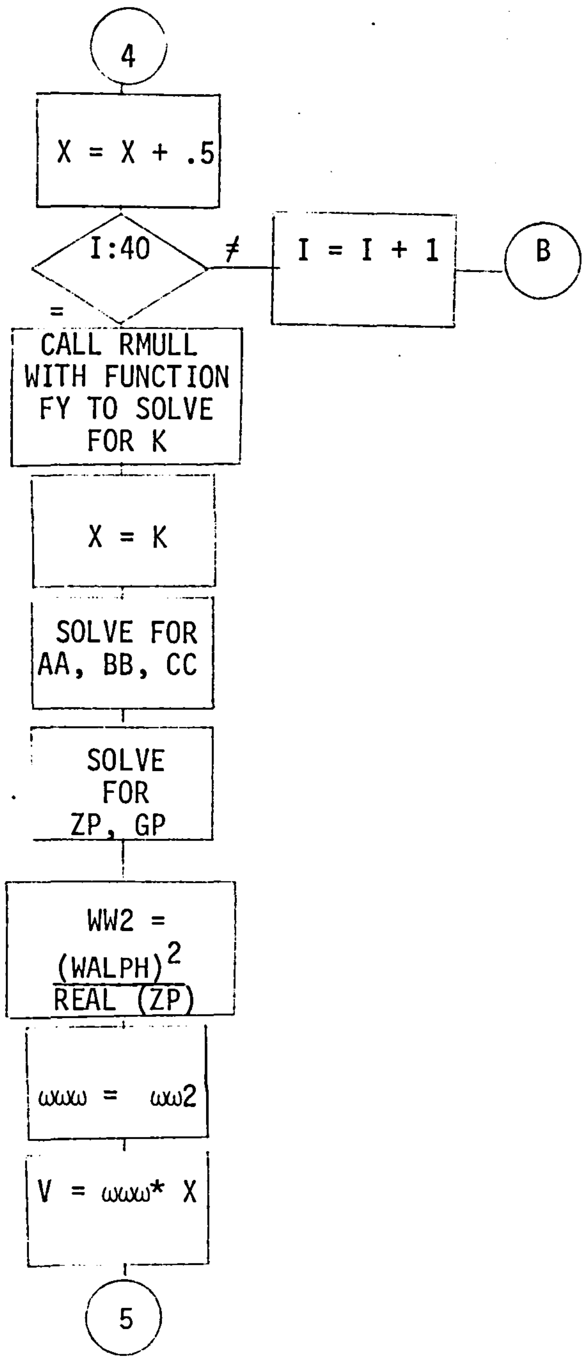
Program Flow Diagram



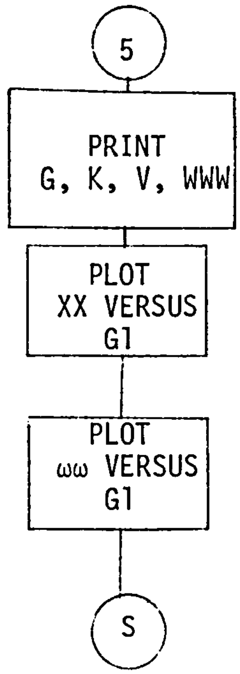

1
SOLVE FOR A1, B1, C1
SOLVE FOR X1, X2
SOLVE FOR A2, B2, C2
X3 = -C2 / B2
G > 0
Y
X3 = (- B2 - (B2)^2 - 4·A2·C2) / 2·A2
N
S X1 = X1
S X2 = X2
S X3 = X3
PRINT X, S X1, S X2, S X3
R2 (I) = S X2
R3 (I) = S X3
X = X + .5
2

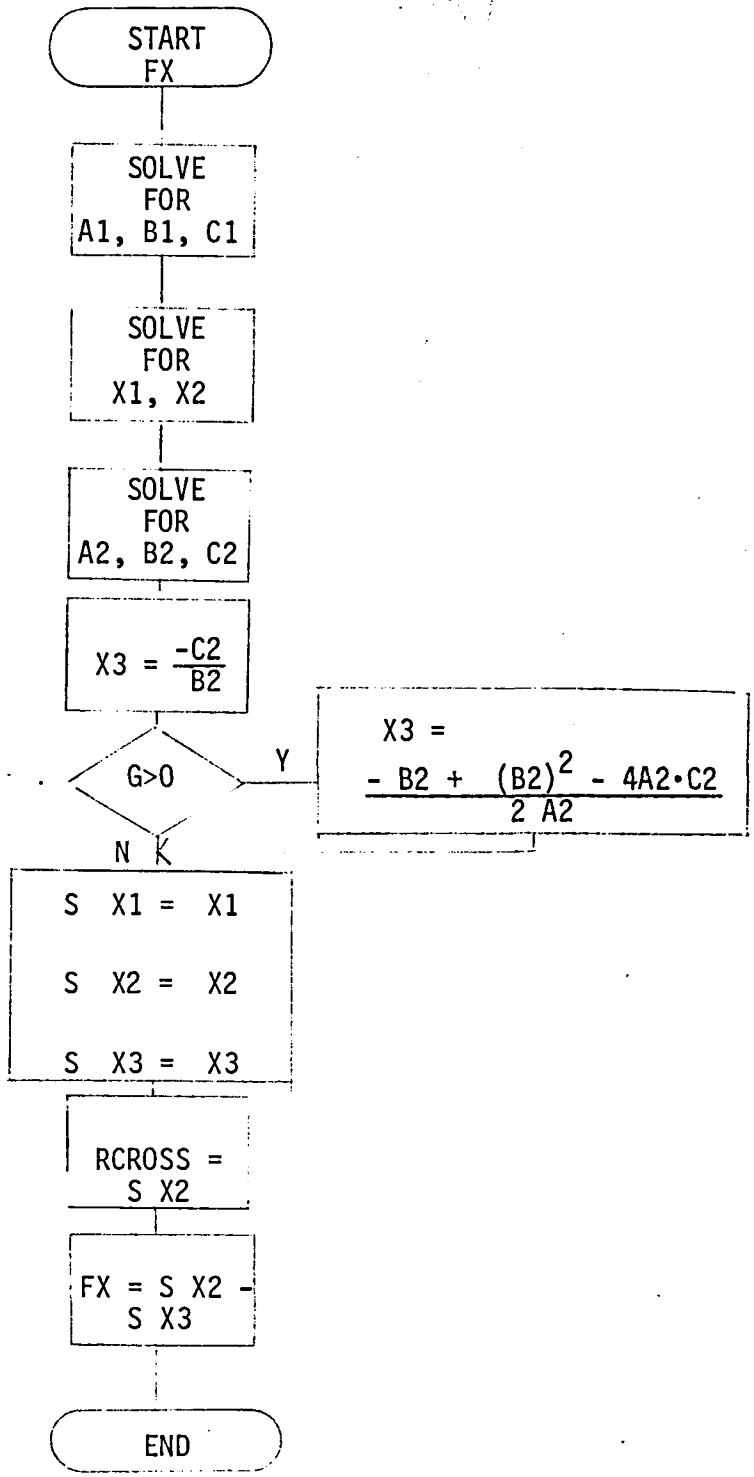

2
I:40
≠
=
I = I + 1
A
CALL RMULL WITH FUNCTION FX TO SOLVE FOR K
PRINT G AND K
PLOT R2, R3 VERSUS XX
PRINT RCROSS
ωωω = WALPH / RCROSS
V = ωωω* K
PRINT ωωω, V
X = 4.0
3

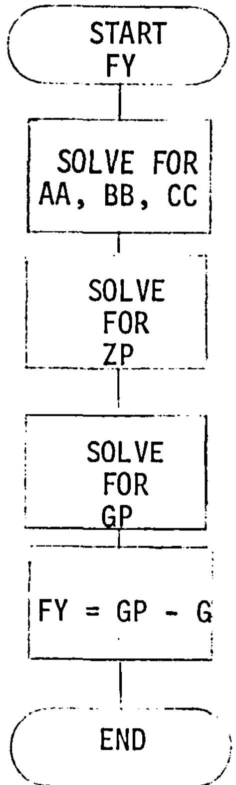

3
PRINT HEADING AND CASE NO.
I = 1
B
SOLVE FOR AA, BB, CC
SOLVE FOR ZP AND ZN
SOLVE FOR GP AND GN
XX (I) = X
PRINT X, ZP, ZN, GP, GN
G1 (I) = GP
WW (I) = REAL (ZP)
4



4
X = X + .5
I:40
≠
I = I + 1
B
=
CALL RMULL WITH FUNCTION FY TO SOLVE FOR K
X = K
SOLVE FOR AA, BB, CC
SOLVE FOR ZP, GP
WW2 = (WALPH)^2 / REAL (ZP)
ωωω = ωω2
V = ωωω* X
5


5
PRINT
G, K, V, WWW
PLOT
XX VERSUS
G1
PLOT
ωω VERSUS
G1
S


START
FX
SOLVE
FOR
A1, B1, C1
SOLVE
FOR
X1, X2
SOLVE
FOR
A2, B2, C2
X3 = -C2 / B2
G>0
Y
N
X3 =
- B2 + (B2)2 - 4A2•C2 / 2 A2
S X1 = X1
S X2 = X2
S X3 = X3
RCROSS =
S X2
FX = S X2 -
S X3
END


START
FY
SOLVE FOR
AA, BB, CC
SOLVE
FOR
ZP
SOLVE
FOR
GP
FY = GP - G
END

SAMPLE PROBLEM

The sample problem presented in this paper is that of a Pylon (strut) that was installed in the NASA Langley Research Center VSTOL tunnel for the purpose of housing wires running from a slip ring assembly through the tunnel wall to the recording instrumentation (see figure 1).

Input Data

The input data for this case is given by the listing on page 33.

Output Data

The output data consists of the tabulation of roots (both real and complex) for given values of 1/K (see pages 34 through 43). Following the roots tabulation the flutter frequency and flutter velocity are printed out. The program plot routine provides the parametric plots for determining the flutter condition from each of the two solution procedures. Plots are given in figures 2 through 4 for a damping value of g = 0 and figures 5 through 7 for g = .01.

The comparison of flutter speeds computed by the two methods and the root locus analysis for g = 0 is given as follows.

	V_F	ω_F
Method 1	10872.9	528.34
Method 2	10919.2	528.34
Root Locus	10878.0	528.0

As can be seen there is excellent agreement between the two methods as you would expect. These solutions are also compared to those obtained by performing a root locus analysis of the roots obtained by Program No. A4813 which also gave excellent agreement.

CONCLUDING REMARKS

This paper has presented two methods for calculating flutter frequency and flutter speed for a system characterized by two degrees of freedom. These methods are simple to apply since the flutter speed and frequency can be solved for directly without resorting to a more complex eigenvalue approach such as a root locus analysis. Program A4868 is available through COSMIC.

APPENDIX A

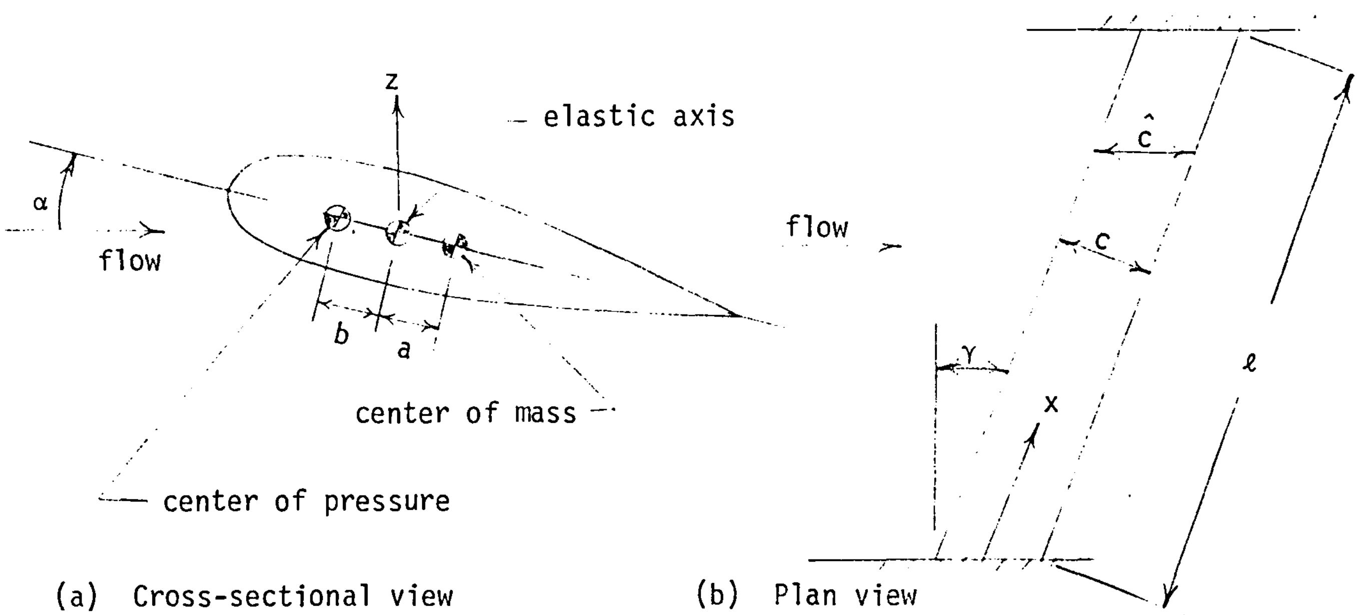
DEVELOPMENT OF GOVERNING EQUATIONS

by

William F. Hunter

The equations of motion are derived for a simple two-degrees-of-freedom representation of the system. The bending and torsional deformations are expressed in terms of assumed displacement functions which satisfy the boundary conditions of the treated problem. The equations are developed for analyzing a strut in the VSTOL wind tunnel; but, with modification, they may be applied to an aircraft wing.

The geometry of the swept strut, which has a uniform cross-section, is shown in the sketch below.



Sketch 1

The displacements z and α are assumed to vary with x according to

$$\frac{z}{c}(x, t) = q_1(t) \sin \frac{\pi x}{\ell}$$

$$\alpha(x, t) = q_2(t) \sin \frac{\pi x}{\ell} \qquad (1)$$

such that q_1 and q_2 become the generalized coordinates of the problem. The kinetic and strain energy are given by

$$T = \frac{1}{2} \int_0^\ell \left[\bar{m} \left(\frac{\partial z_{cm}}{\partial t}\right)^2 + \bar{I}_{cm} \left(\frac{\partial \alpha}{\partial t}\right)^2 \right] d x$$

$$U = \frac{1}{2} \int_0^\ell \left[EI \left(\frac{\partial^2 z}{\partial x^2}\right)^2 + G J \left(\frac{\partial \alpha}{\partial x}\right)^2 \right] d x \qquad (2)$$

where $\bar{m}$ is the distributed mass, $\bar{I}_{cm}$ is the distributed mass moment of inertia about the center of mass axis, EI is the bending stiffness, and GJ is the torsional stiffness. Noting that $z_{cm} = z - a\,\alpha$ and letting $m = \bar{m}\,\ell$ and $\tilde{I} = \bar{I}_{cm}\,\ell$, the kinetic and strain energy may be expressed in quadratic form as

$$T = \frac{1}{2} \{\dot{q}\}^T [M] \{\dot{q}\}$$

$$U = \frac{1}{2} \{q\} \; [K] \{q\} \qquad (3)$$

where the mass and stiffness matrices are given by

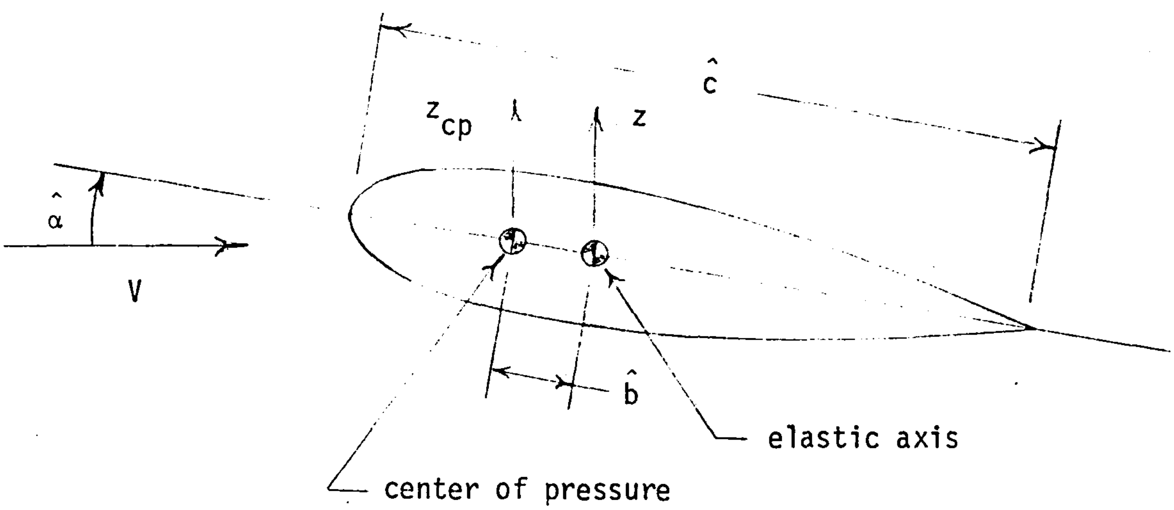
$$[M] = \frac{m}{2} \begin{bmatrix} c^2 & -ac \\ -ac & a^2 + \frac{\tilde{I}}{m} \end{bmatrix} \tag{4}$$

$$[K] = \frac{\pi^2}{2\ell} \begin{bmatrix} \left(\frac{\pi c}{\ell}\right)^2 EI & 0 \\ 0 & GJ \end{bmatrix} \tag{5}$$

and

$$\{q\} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{6}$$

Expressions for the generalized aerodynamic forces are obtained by considering a streamwise cross-section (see sketch 2) rather the previously depicted section which was taken normal to the x-axis.



Sketch 2

Note that

$$\hat{c} = c/\cos\gamma$$

$$\hat{b} = b/\cos\gamma \qquad (7)$$

$$\hat{x} = x\cos\gamma$$

$$\hat{\alpha} = \alpha\cos\gamma$$

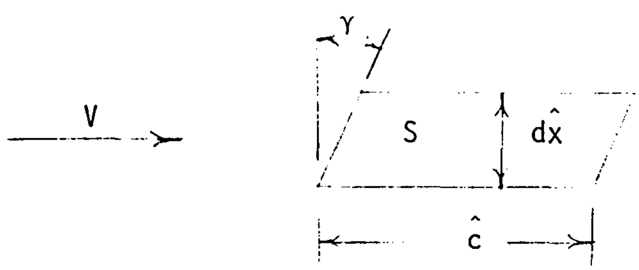
Since the velocity of the center of pressure in the direction of z is

$$\dot{z}_{cp} = \dot{z} + \hat{b}\,\dot{\hat{\alpha}}$$

the total angle of attack is

$$\hat{\alpha}_t = \hat{\alpha} - \dot{z}_{cp}/V = \hat{\alpha} - \dot{z}/V - \hat{b}\;\dot{\hat{\alpha}}/V \qquad (8)$$

The aerodynamic loadings on the differential segment



are approximated using simple strip theory by

$$L = \frac{1}{2} \rho V^2 S C_L$$
$$M = \frac{1}{2} \rho V^2 S C_M \qquad (9)$$

where

$$C_L = C_{L_\alpha} \hat{\alpha}_t$$
$$C_M = C_L \hat{b} \qquad (10)$$
$$S = \hat{c} d \hat{x} = cdx$$

The virtual work of the aerodynamic forces acting on the differential segment is given by

$$\overline{\delta W} = L \delta_{\dot{z}} + M \delta \hat{\alpha} \qquad (11)$$

substituting equations (1), (7), (8), (9), and (10) into equations (11) gives

$$\overline{\delta W} = \frac{1}{2} \rho V^2 c C_{L_\alpha} (q_2 \cos \gamma - \frac{c}{V} \dot{q}_1 - \frac{b}{V} \dot{q}_2) (c \delta_{q_1} + b \delta_{q_2}) \sin^2 (\frac{\pi x}{\ell}) dx \qquad (12)$$

The virtual work for the entire strut is given by

$$\delta W = \int_o^\ell \overline{\delta W}$$

Integrating and comparing the result with $\delta W = \Sigma Q_i \delta_{q_i}$ defines the generalized forces Q_1 and Q_2. These forces may be expressed in a matrix equation as

$$\{Q\} = V^2 [D] \{q\} - V [R] \{\dot{q}\} \tag{13}$$

where the aerodynamics matrices [D] and [R] associated with the displacements and rates, respectively, are

$$[D] = \frac{1}{4} \rho \; c \; \ell \; C_{L_\alpha} \begin{bmatrix} 0 & c \cos \gamma \\ 0 & b \cos \gamma \end{bmatrix} \tag{14}$$

$$[R] = \frac{1}{4} \rho \; c \; \ell \; C_{L_\alpha} \begin{bmatrix} c^2 & b\,c \\ b\,c & b^2 \end{bmatrix} \tag{15}$$

and

$$\{Q\} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} . \tag{16}$$

Now that the kinetic energy, strain energy, and generalized forces have been expressed, Lagrange's equations yield

$$[M] \{\ddot{q}\} + [K] \{q\} = \{Q\} \tag{17}$$

Introducing structural damping g and substituting for {Q} gives the governing differential equation as

$$[M] \{\ddot{q}\} + (1 + ig) [K] \{q\} - V^2 [D] \{q\} + V [R] \{\dot{q}\} = \{0\} \tag{18}$$

REFERENCES

1. Fung, Y. C.: An Introduction to the Theory of Aeroelasticity. John Wiley and Sons, Incorporated, c. 1955, pp. 235-242.

PARTIAL PROGRAM LISTING

```
      PROGRAM FDETER(INPUT,OUTPUT,TAPE5,TAPE6)
C     C  PINNED-PINNED CASE
      REAL J,M(2,2),KK(2,2)
      DIMENSION R(2,2), DD(2,2)
      DIMENSION R2(42), R3(42), XX(42)
      DIMENSION EPS(3)
      REAL K
      COMPLEX ZP,ZN,BB,CC
      DIMENSION G1(42), WW(42)
      COMMON /BLK1/ A,B,C,D,E,F,H,J,W,G
      COMMON /BLK2/ RCROSS
      EXTERNAL FY
      EXTERNAL FX
      NAMELIST /NAM2/ G,WALPH,WY
      NAMELIST /NAM1/ M,DD,R
      DATA EPS/3*.001/
      CALL PSEUDO
      ICASE=0
1     READ (5,NAM1)
      IF (EOF(5)) 5,2
2     READ (5,NAM2)
      ICASE=ICASE+1
      A=R(1,1)/M(1,1)
      B=M(1,2)/M(1,1)
      C=DD(1,2)/M(1,1)
      D=R(1,2)/M(1,1)
      E=M(2,1)/M(2,2)
      F=R(2,1)/M(2,2)
      H=DD(2,2)/M(2,2)
      J=R(2,2)/M(2,2)
      W=(WY/WALPH)
      X=4.0
      WRITE(6,11)ICASE
      DO 3 I=1,40
C     X=1/K
      AA=W**2
      S=-(W**2*(1.-H*X**2)-1.)
      T=-(A*X+W**2*J*X)
```

```
      BB=CMPLX(S,T)
      U=(1.+H*X**2-A*J*X**2-E*B-C*E*X**2+D*F*X**2)
      V=(X*(-A-A*H*X**2-J+E*D+B*F+C*F*X**2))
      CC=CMPLX(U,V)
      ZP=(-BB+CSQRT(BB**2-4.*AA*CC))/(2.*AA)
      ZN=(-BB-CSQRT(BB**2-4.*AA*CC))/(2.*AA)
      GP=(AIMAG(ZP))/(REAL(ZP))
      GN=(AIMAG(ZN))/(REAL(ZN))
      XX(I)=X
      WRITE(6,12)X,ZP,ZN,GP,GN
      G1(I)=GP
      WW(I)=REAL(ZP)
      X=X+.5
3     CONTINUE
      CALL RMULL (FY,2.,.5,EPS,K,NUM,IERR)
      X=K
      AA=W**2
      S=-(W**2*(1.-H*X**2)-1.)
      T=-(A*X+W**2*J*X)
      BB=CMPLX(S,T)
      U=(1.+H*X**2-A*J*X**2-E*B-C*E*X**2+D*F*X**2)
      V=(X*(-A-A*H*X**2-J+E*D+B*F+C*F*X**2))
      CC=CMPLX(U,V)
      ZP=(-BB+CSQRT(BB**2-4.*AA*CC))/(2.*AA)
      GP=(AIMAG(ZP))/(REAL(ZP))
      WW2=WALPH**2/(REAL(ZP))
      WWW=SQRT(ABS(WW2))
      V=WWW*X
      WRITE(6,13)G,K
      WRITE(6,10)WWW,V
      CALL PLOT (XX,G1,1,2,1)
      CALL PLOT (WW,G1,3,2,2)
      X=4.0
      WRITE(6,6)ICASE
      DO 4 I=1,40
      XX(I)=X
C     X=1/K
      A1=W**2*(1.-G**2)
```

PARTIAL PROGRAM LISTING (CONT'D)

```
      B1=(-1.-W**2*(1.+X**2*H+G*J*X))
      C1=1.+X**2*H-X**2*A*J-E*B-E*C*X**2+F*D*X**2
      X1=(-B1+SQRT(B1**2-4.*A1*C1))/(2.*A1)
      X2=(-B1-SQRT(B1**2-4.*A1*C1))/(2.*A1)
      A2=2.*W*G
      B2=(-G-W**2*(G+X**2*H*G-X*J)+A*X+A*X*G)
      C2=X*(-A-A*H*X**2-J+E*D+F*B+F*C*X**2)
      X3=-C2/B2
      IF (G.GT.0.) X3=(-B2+SQRT(B2**2-4.*A2*C2))/(2.*A2)
      SX1=SQRT(X1)
      SX2=SQRT(X2)
      SX3=SQRT(X3)
      WRITE(6,7)X,SX1,SX2,SX3
      R2(I)=SX2
      R3(I)=SX3
      X=X+.5
4     CONTINUE
      CALL RMULL (FX,2.,.5,EPS,K,NUM,IERR)
      WRITE(6,8)G,K
      CALL PLOT1 (R2,R3,XX)
      WRITE(6,9)RCROSS
      WWW=WALPH/RCROSS
      V=WWW*K
      WRITE(6,10)WWW,V
      GO TO 1
5     CALL CALPLT (0.0,0.0,999.)
      STOP
C
C
C
6     FORMAT (1H1,47H ROOTS OF REAL AND IMAGINARY EQUATIONS FOR CASE,I2)
7     FORMAT (/,5H 1/K=,F6.3,3X,7HSQRTX1=,F15.6,3X,7HSQRTX2=,F15.6,3X,8H
     1 SQRTX3=,F15.6)
8     FORMAT (//,3H G=,F5.2,9H FOR 1/K=,F8.4)
9     FORMAT (/,58H REAL AND IMAGINARY ROOTS INTERSECT WHEN SQUARE ROOT
     1OF X=,F8.4)
10    FORMAT (/,3H W=,F12.4,2X,2HV=,F12.4)
11    FORMAT (1H1,46H COMPLEX ROOTS OF FLUTTER DETERMINANT FOR CASE,I2)
```

```
12    FORMAT (//,5H 1/K=,F6.3,3X,3HZP=,F15.6,1X,F15.6,3X,3HZN=,F15.6,1X,
     1F15.6,3X,3HGP=,F15.6,3X,3HGN=,F15.6)
13    FORMAT (//,3H G=,F5.2,9H FOR 1/K=,F8.4)
      END
      FUNCTION FX (X)
      COMMON /BLK2/ RCROSS
      REAL J
      COMMON /BLK1/ A,B,C,D,E,F,H,J,W,G
      A1=W**2*(1.-G**2)
      B1=(-1.-W**2*(1.+X**2*H+G*J*X))
      C1=1.+X**2*H-X**2*A*J-E*B-E*C*X**2+F*D*X**2
      X1=(-B1+SQRT(B1**2-4.*A1*C1))/(2.*A1)
      X2=(-B1-SQRT(B1**2-4.*A1*C1))/(2.*A1)
      A2=2.*W*G
      B2=(-G-W**2*(G+X**2*H*G-X*J)+A*X+A*X*G)
      C2=X*(-A-A*H*X**2-J+E*D+F*B+F*C*X**2)
      X3=-C2/B2
      IF (G.GT.0.) X3=(-B2+SQRT(B2**2-4.*A2*C2))/(2.*A2)
      SX1=SQRT(X1)
      SX2=SQRT(X2)
      RCROSS=SX2
      SX3=SQRT(X3)
      FX=SX2-SX3
      RETURN
      END
      FUNCTION FY (X)
      COMMON /BLK1/ A,B,C,D,E,F,H,J,W,G
      COMPLEX BB,CC,ZP
      REAL J
      AA=W**2
      S=-(W**2*(1.-H*X**2)-1.)
      T=-(A*X+W**2*J*X)
      BB=CMPLX(S,T)
      U=(1.+H*X**2-A*J*X**2-E*B-C*E*X**2+D*F*X**2)
      V=(X*(-A-A*H*X**2-J+E*D+B*F+C*F*X**2))
      CC=CMPLX(U,V)
      ZP=(-BB+CSQRT(BB**2-4.*AA*CC))/(2.*AA)
      GP=(AIMAG(ZP))/(REAL(ZP))
      FY=GP-G
      RETURN
```

INPUT DATA FOR SAMPLE PROBLEM
CASE 1

```
$NAM1 W=11.302,-1.05,-1.05,1.07,DD=0.,0.,.0051,0.00051,R=.056,.0056,.0055,
.000565
$NAM2 G=0.,WALPH=501.52,WY=21.37$
```

INPUT DATA FOR SAMPLE PROBLEM
CASE 2

```
$NAM1 W=11.302,-1.05,-1.05,1.07,DD=0.,0.,.0051,0.00051,R=.056,.0056,.0056,
.000565
$NAM2 G=.01,WALPH=501.52,WY=21.37$
```

OUTPUT DATA FOR SAMPLE PROBLEM

COMPLEX ROOTS OF FLUTTER DETERMINANT FOR CASE 1

1/K= 4.000	ZP=	-.927339	.007338	ZN=	-790.382780	15.697760	GP=	-.007912	GN=	-.019861
1/K= 4.500	ZP=	-.931223	.008168	ZN=	-790.380885	17.660067	GP=	-.008771	GN=	-.022344
1/K= 5.000	ZP=	-.935564	.008968	ZN=	-790.378767	19.622405	GP=	-.009585	GN=	-.024827
1/K= 5.500	ZP=	-.940360	.009733	ZN=	-790.376427	21.584776	GP=	-.010351	GN=	-.027309
1/K= 6.000	ZP=	-.945611	.010461	ZN=	-790.373866	23.547185	GP=	-.011063	GN=	-.029792
1/K= 6.500	ZP=	-.951317	.011149	ZN=	-790.371084	25.509635	GP=	-.011719	GN=	-.032276
1/K= 7.000	ZP=	-.957478	.011792	ZN=	-790.368081	27.472129	GP=	-.012316	GN=	-.034759
1/K= 7.500	ZP=	-.964094	.012388	ZN=	-790.364858	29.434671	GP=	-.012849	GN=	-.037242
1/K= 8.000	ZP=	-.971162	.012933	ZN=	-790.361416	31.397263	GP=	-.013317	GN=	-.039725
1/K= 8.500	ZP=	-.978684	.013423	ZN=	-790.357754	33.359910	GP=	-.013715	GN=	-.042209
1/K= 9.000	ZP=	-.986659	.013856	ZN=	-790.353874	35.322614	GP=	-.014043	GN=	-.044692
1/K= 9.500	ZP=	-.995085	.014228	ZN=	-790.349775	37.285380	GP=	-.014298	GN=	-.047176
1/K=10.000	ZP=	-1.003963	.014535	ZN=	-790.345459	39.248209	GP=	-.014478	GN=	-.049660
1/K=10.500	ZP=	-1.013292	.014775	ZN=	-790.340926	41.211107	GP=	-.014581	GN=	-.052143
1/K=11.000	ZP=	-1.023071	.014944	ZN=	-790.336177	43.174075	GP=	-.014607	GN=	-.054627
1/K=11.500	ZP=	-1.033299	.015038	ZN=	-790.331213	45.137118	GP=	-.014554	GN=	-.057112

OUTPUT DATA FOR SAMPLE PROBLEM (CONT'D)

1/K=12.000	ZP=	-1.043976	.015055	ZN=	-790.326033	47.100239	GP=	-.014421	GN=	-.059596
1/K=12.500	ZP=	-1.055101	.014991	ZN=	-790.320640	49.063440	GP=	-.014208	GN=	-.062080
1/K=13.000	ZP=	-1.066673	.014842	ZN=	-790.315034	51.026726	GP=	-.013915	GN=	-.064565
1/K=13.500	ZP=	-1.078691	.014606	ZN=	-790.309215	52.990099	GP=	-.013541	GN=	-.067050
1/K=14.000	ZP=	-1.091155	.014280	ZN=	-790.303185	54.953563	GP=	-.013087	GN=	-.069535
1/K=14.500	ZP=	-1.104063	.013859	ZN=	-790.296944	56.917121	GP=	-.012553	GN=	-.072020
1/K=15.000	ZP=	-1.117415	.013341	ZN=	-790.290494	58.880776	GP=	-.011939	GN=	-.074505
1/K=15.500	ZP=	-1.131209	.012722	ZN=	-790.283835	60.844532	GP=	-.011246	GN=	-.076991
1/K=16.000	ZP=	-1.145445	.011999	ZN=	-790.276968	62.808392	GP=	-.010476	GN=	-.079476
1/K=16.500	ZP=	-1.160122	.011170	ZN=	-790.269894	64.772359	GP=	-.009628	GN=	-.081962
1/K=17.000	ZP=	-1.175238	.010230	ZN=	-790.262615	66.736436	GP=	-.008705	GN=	-.084448
1/K=17.500	ZP=	-1.190793	.009177	ZN=	-790.255132	68.700626	GP=	-.007706	GN=	-.086935
1/K=18.000	ZP=	-1.206785	.008007	ZN=	-790.247444	70.664934	GP=	-.006635	GN=	-.089421
1/K=18.500	ZP=	-1.223214	.006717	ZN=	-790.239555	72.629360	GP=	-.005491	GN=	-.091908
1/K=19.000	ZP=	-1.240077	.005304	ZN=	-790.231464	74.593910	GP=	-.004278	GN=	-.094395
1/K=19.500	ZP=	-1.257375	.003766	ZN=	-790.223174	76.558586	GP=	-.002995	GN=	-.096882
1/K=20.000	ZP=	-1.275105	.002098	ZN=	-790.214684	78.523392	GP=	-.001645	GN=	-.099370

OUTPUT DATA FOR SAMPLE PROBLEM (CONT'D)

1/K=20.500	ZP=	-1.293266	.000297	ZN=	-790.205998	80.488329	GP=	-.000230	GN=	-.101857
1/K=21.000	ZP=	-1.311858	-.001639	ZN=	-790.197115	82.453402	GP=	.001249	GN=	-.104345
1/K=21.500	ZP=	-1.330878	-.003713	ZN=	-790.188037	84.418614	GP=	.002790	GN=	-.106834
1/K=22.000	ZP=	-1.350326	-.005929	ZN=	-790.178766	86.383967	GP=	.004391	GN=	-.109322
1/K=22.500	ZP=	-1.370200	-.008290	ZN=	-790.169302	88.349465	GP=	.006050	GN=	-.111811
1/K=23.000	ZP=	-1.390499	-.010798	ZN=	-790.159648	90.315111	GP=	.007766	GN=	-.114300
1/K=23.500	ZP=	-1.411221	-.013458	ZN=	-790.149804	92.280908	GP=	.009536	GN=	-.116789

G= 0.00 FOR 1/K= 20.5791

W= 528.3450 V= 10872.8658

OUTPUT SAMPLE PROBLEM

ROOTS OF REAL AND IMAGINARY EQUATIONS FOR CASE 1

1/K=	4.000	SQRTX1=	28.149339	SQRTX2=	.961691	SQRTX3=	1.138499
1/K=	4.500	SQRTX1=	28.149306	SQRTX2=	.963685	SQRTX3=	1.138499
1/K=	5.000	SQRTX1=	28.149269	SQRTX2=	.965907	SQRTX3=	1.138499
1/K=	5.500	SQRTX1=	28.149229	SQRTX2=	.968358	SQRTX3=	1.138499
1/K=	6.000	SQRTX1=	28.149184	SQRTX2=	.971036	SQRTX3=	1.138499
1/K=	6.500	SQRTX1=	28.149136	SQRTX2=	.973937	SQRTX3=	1.138499
1/K=	7.000	SQRTX1=	28.149084	SQRTX2=	.977062	SQRTX3=	1.138499
1/K=	7.500	SQRTX1=	28.149028	SQRTX2=	.980406	SQRTX3=	1.138499
1/K=	8.000	SQRTX1=	28.148968	SQRTX2=	.983969	SQRTX3=	1.138499
1/K=	8.500	SQRTX1=	28.148904	SQRTX2=	.987747	SQRTX3=	1.138499
1/K=	9.000	SQRTX1=	28.148836	SQRTX2=	.991739	SQRTX3=	1.138499
1/K=	9.500	SQRTX1=	28.148765	SQRTX2=	.995942	SQRTX3=	1.138499
1/K=	10.000	SQRTX1=	28.148689	SQRTX2=	1.000352	SQRTX3=	1.138499
1/K=	10.500	SQRTX1=	28.148610	SQRTX2=	1.004968	SQRTX3=	1.138499
1/K=	11.000	SQRTX1=	28.148527	SQRTX2=	1.009787	SQRTX3=	1.138499
1/K=	11.500	SQRTX1=	28.148440	SQRTX2=	1.014805	SQRTX3=	1.138499
1/K=	12.000	SQRTX1=	28.148349	SQRTX2=	1.020020	SQRTX3=	1.138499
1/K=	12.500	SQRTX1=	28.148255	SQRTX2=	1.025429	SQRTX3=	1.138499
1/K=	13.000	SQRTX1=	28.148156	SQRTX2=	1.031028	SQRTX3=	1.138499
1/K=	13.500	SQRTX1=	28.148054	SQRTX2=	1.036815	SQRTX3=	1.138499
1/K=	14.000	SQRTX1=	28.147947	SQRTX2=	1.042787	SQRTX3=	1.138499
1/K=	14.500	SQRTX1=	28.147837	SQRTX2=	1.048940	SQRTX3=	1.138499
1/K=	15.000	SQRTX1=	28.147723	SQRTX2=	1.055271	SQRTX3=	1.138499
1/K=	15.500	SQRTX1=	28.147605	SQRTX2=	1.061777	SQRTX3=	1.138499

OUTPUT SAMPLE PROBLEM (CONT'D)

1/K=16.000	SQRTX1=	28.147483	SQRTX2=	1.068455	SQRTX3=	1.138499
1/K=16.500	SQRTX1=	28.147358	SQRTX2=	1.075302	SQRTX3=	1.138499
1/K=17.000	SQRTX1=	28.147228	SQRTX2=	1.082314	SQRTX3=	1.138499
1/K=17.500	SQRTX1=	28.147095	SQRTX2=	1.089489	SQRTX3=	1.138499
1/K=18.000	SQRTX1=	28.146957	SQRTX2=	1.096822	SQRTX3=	1.138499
1/K=18.500	SQRTX1=	28.146816	SQRTX2=	1.104312	SQRTX3=	1.138499
1/K=19.000	SQRTX1=	28.146671	SQRTX2=	1.111954	SQRTX3=	1.138499
1/K=19.500	SQRTX1=	28.146522	SQRTX2=	1.119746	SQRTX3=	1.138499
1/K=20.000	SQRTX1=	28.146370	SQRTX2=	1.127684	SQRTX3=	1.138499
1/K=20.500	SQRTX1=	28.146213	SQRTX2=	1.135767	SQRTX3=	1.138499
1/K=21.000	SQRTX1=	28.146052	SQRTX2=	1.143989	SQRTX3=	1.138499
1/K=21.500	SQRTX1=	28.145888	SQRTX2=	1.152349	SQRTX3=	1.138499
1/K=22.000	SQRTX1=	28.145720	SQRTX2=	1.160844	SQRTX3=	1.138499
1/K=22.500	SQRTX1=	28.145547	SQRTX2=	1.169471	SQRTX3=	1.138499
1/K=23.000	SQRTX1=	28.145371	SQRTX2=	1.178226	SQRTX3=	1.138499
1/K=23.500	SQRTX1=	28.145191	SQRTX2=	1.187107	SQRTX3=	1.138499

G= 0.00 FOR 1/K= 20.6671

REAL AND IMAGINARY ROOTS INTERSECT WHEN SQUARE ROOT OF X= 1.1385

W= 528.3405 V= 10919.2489

OUTPUT DATA FOR SAMPLE PROBLEM

COMPLEX ROOTS OF FLUTTER DETERMINANT FOR CASE 2

1/K=	4.000	ZP=	-.927339	.007338	ZN=	-790.382780	15.697760	GP=	-.007912	GN=	-.019861
1/K=	4.500	ZP=	-.931223	.008168	ZN=	-790.380885	17.660067	GP=	-.008771	GN=	-.022344
1/K=	5.000	ZP=	-.935564	.008968	ZN=	-790.378767	19.622405	GP=	-.009585	GN=	-.024827
1/K=	5.500	ZP=	-.940360	.009733	ZN=	-790.376427	21.584776	GP=	-.010351	GN=	-.027309
1/K=	6.000	ZP=	-.945611	.010461	ZN=	-790.373866	23.547185	GP=	-.011063	GN=	-.029792
1/K=	6.500	ZP=	-.951317	.011149	ZN=	-790.371084	25.509635	GP=	-.011719	GN=	-.032276
1/K=	7.000	ZP=	-.957478	.011792	ZN=	-790.368081	27.472129	GP=	-.012316	GN=	-.034759
1/K=	7.500	ZP=	-.964094	.012388	ZN=	-790.364858	29.434671	GP=	-.012849	GN=	-.037242
1/K=	8.000	ZP=	-.971162	.012933	ZN=	-790.361416	31.397263	GP=	-.013317	GN=	-.039725
1/K=	8.500	ZP=	-.978684	.013423	ZN=	-790.357754	33.359910	GP=	-.013715	GN=	-.042209
1/K=	9.000	ZP=	-.986659	.013856	ZN=	-790.353874	35.322614	GP=	-.014043	GN=	-.044692
1/K=	9.500	ZP=	-.995085	.014228	ZN=	-790.349775	37.285380	GP=	-.014298	GN=	-.047176
1/K=	10.000	ZP=	-1.003963	.014535	ZN=	-790.345459	39.248209	GP=	-.014478	GN=	-.049660
1/K=	10.500	ZP=	-1.013292	.014775	ZN=	-790.340926	41.211107	GP=	-.014581	GN=	-.052143
1/K=	11.000	ZP=	-1.023071	.014944	ZN=	-790.336177	43.174075	GP=	-.014607	GN=	-.054627
1/K=	11.500	ZP=	-1.033299	.015038	ZN=	-790.331213	45.137118	GP=	-.014554	GN=	-.057112

OUTPUT DATA FOR SAMPLE PROBLEM (CONT'D)

1/K=12.000	ZP=	-1.043976	.015055	ZN=	-790.326033	47.100239	GP=	-.014421	GN=	-.059596
1/K=12.500	ZP=	-1.055101	.014991	ZN=	-790.320640	49.063440	GP=	-.014208	GN=	-.062080
1/K=13.000	ZP=	-1.066673	.014842	ZN=	-790.315034	51.026726	GP=	-.013915	GN=	-.064565
1/K=13.500	ZP=	-1.078691	.014606	ZN=	-790.309215	52.990099	GP=	-.013541	GN=	-.067050
1/K=14.000	ZP=	-1.091155	.014280	ZN=	-790.303185	54.953563	GP=	-.013087	GN=	-.069535
1/K=14.500	ZP=	-1.104063	.013859	ZN=	-790.296944	56.917121	GP=	-.012553	GN=	-.072020
1/K=15.000	ZP=	-1.117415	.013341	ZN=	-790.290494	58.880776	GP=	-.011939	GN=	-.074505
1/K=15.500	ZP=	-1.131209	.012722	ZN=	-790.283835	60.844532	GP=	-.011246	GN=	-.076991
1/K=16.000	ZP=	-1.145445	.011999	ZN=	-790.276968	62.808392	GP=	-.010476	GN=	-.079476
1/K=16.500	ZP=	-1.160122	.011170	ZN=	-790.269894	64.772359	GP=	-.009628	GN=	-.081962
1/K=17.000	ZP=	-1.175238	.010230	ZN=	-790.262615	66.736436	GP=	-.008705	GN=	-.084448
1/K=17.500	ZP=	-1.190793	.009177	ZN=	-790.255132	68.700626	GP=	-.007706	GN=	-.086935
1/K=18.000	ZP=	-1.206785	.008007	ZN=	-790.247444	70.664934	GP=	-.006635	GN=	-.089421
1/K=18.500	ZP=	-1.223214	.006717	ZN=	-790.239555	72.629360	GP=	-.005491	GN=	-.091908
1/K=19.000	ZP=	-1.240077	.005304	ZN=	-790.231464	74.593910	GP=	-.004278	GN=	-.094395
1/K=19.500	ZP=	-1.257375	.003766	ZN=	-790.223174	76.558586	GP=	-.002995	GN=	-.096882
1/K=20.000	ZP=	-1.275105	.002098	ZN=	-790.214684	78.523392	GP=	-.001645	GN=	-.099370

OUTPUT DATA FOR SAMPLE PROBLEM (CONT'D)

1/K=20.500	ZP=	-1.293266	.000297	ZN=	-790.205998	80.488329	GP=	-.000230	GN=	-.101857
1/K=21.000	ZP=	-1.311858	-.001639	ZN=	-790.197115	82.453402	GP=	.001249	GN=	-.104345
1/K=21.500	ZP=	-1.330878	-.003713	ZN=	-790.188037	84.418614	GP=	.002790	GN=	-.106834
1/K=22.000	ZP=	-1.350326	-.005929	ZN=	-790.178766	86.383967	GP=	.004391	GN=	-.109322
1/K=22.500	ZP=	-1.370200	-.008290	ZN=	-790.169302	88.349465	GP=	.006050	GN=	-.111811
1/K=23.000	ZP=	-1.390499	-.010798	ZN=	-790.159648	90.315111	GP=	.007766	GN=	-.114300
1/K=23.500	ZP=	-1.411221	-.013458	ZN=	-790.149804	92.280908	GP=	.009536	GN=	-.116789

G= .01 FOR 1/K= 23.6285

W= 505.3868 V= 11941.5551

OUTPUT DATA FOR SAMPLE PROBLEM (CONT'D)

ROOTS OF REAL AND IMAGINARY EQUATIONS FOR CASE 2

1/K= 4.000	SQRTX1=	28.150748	SQRTX2=	.961691	SQRTX3=	1.489313
1/K= 4.500	SQRTX1=	28.150715	SQRTX2=	.963684	SQRTX3=	1.437943
1/K= 5.000	SQRTX1=	28.150679	SQRTX2=	.965907	SQRTX3=	1.399236
1/K= 5.500	SQRTX1=	28.150638	SQRTX2=	.968358	SQRTX3=	1.369109
1/K= 6.000	SQRTX1=	28.150594	SQRTX2=	.971036	SQRTX3=	1.345031
1/K= 6.500	SQRTX1=	28.150546	SQRTX2=	.973937	SQRTX3=	1.325365
1/K= 7.000	SQRTX1=	28.150494	SQRTX2=	.977062	SQRTX3=	1.309012
1/K= 7.500	SQRTX1=	28.150438	SQRTX2=	.980406	SQRTX3=	1.295204
1/K= 8.000	SQRTX1=	28.150378	SQRTX2=	.983969	SQRTX3=	1.283393
1/K= 8.500	SQRTX1=	28.150314	SQRTX2=	.987747	SQRTX3=	1.273177
1/K= 9.000	SQRTX1=	28.150246	SQRTX2=	.991739	SQRTX3=	1.264256
1/K= 9.500	SQRTX1=	28.150175	SQRTX2=	.995942	SQRTX3=	1.256397
1/K=10.000	SQRTX1=	28.150100	SQRTX2=	1.000352	SQRTX3=	1.249424
1/K=10.500	SQRTX1=	28.150020	SQRTX2=	1.004968	SQRTX3=	1.243194
1/K=11.000	SQRTX1=	28.149937	SQRTX2=	1.009787	SQRTX3=	1.237595
1/K=11.500	SQRTX1=	28.149851	SQRTX2=	1.014805	SQRTX3=	1.232535
1/K=12.000	SQRTX1=	28.149760	SQRTX2=	1.020020	SQRTX3=	1.227942
1/K=12.500	SQRTX1=	28.149665	SQRTX2=	1.025429	SQRTX3=	1.223752
1/K=13.000	SQRTX1=	28.149567	SQRTX2=	1.031028	SQRTX3=	1.219916
1/K=13.500	SQRTX1=	28.149464	SQRTX2=	1.036815	SQRTX3=	1.216390
1/K=14.000	SQRTX1=	28.149358	SQRTX2=	1.042787	SQRTX3=	1.213138
1/K=14.500	SQRTX1=	28.149248	SQRTX2=	1.048940	SQRTX3=	1.210130
1/K=15.000	SQRTX1=	28.149134	SQRTX2=	1.055271	SQRTX3=	1.207339
1/K=15.500	SQRTX1=	28.149016	SQRTX2=	1.061777	SQRTX3=	1.204742

OUTPUT DATA FOR SAMPLE PROBLEM (CONT'D)

1/K=16.000	SQRTX1=	28.148894	SQRTX2=	1.068455	SQRTX3=	1.202321
1/K=16.500	SQRTX1=	28.148769	SQRTX2=	1.075302	SQRTX3=	1.200057
1/K=17.000	SQRTX1=	28.148639	SQRTX2=	1.082314	SQRTX3=	1.197936
1/K=17.500	SQRTX1=	28.148506	SQRTX2=	1.089488	SQRTX3=	1.195944
1/K=18.000	SQRTX1=	28.148369	SQRTX2=	1.096822	SQRTX3=	1.194071
1/K=18.500	SQRTX1=	28.148228	SQRTX2=	1.104311	SQRTX3=	1.192306
1/K=19.000	SQRTX1=	28.148083	SQRTX2=	1.111954	SQRTX3=	1.190640
1/K=19.500	SQRTX1=	28.147934	SQRTX2=	1.119746	SQRTX3=	1.189065
1/K=20.000	SQRTX1=	28.147781	SQRTX2=	1.127684	SQRTX3=	1.187573
1/K=20.500	SQRTX1=	28.147624	SQRTX2=	1.135766	SQRTX3=	1.186159
1/K=21.000	SQRTX1=	28.147464	SQRTX2=	1.143989	SQRTX3=	1.184815
1/K=21.500	SQRTX1=	28.147300	SQRTX2=	1.152349	SQRTX3=	1.183538
1/K=22.000	SQRTX1=	28.147131	SQRTX2=	1.160844	SQRTX3=	1.182322
1/K=22.500	SQRTX1=	28.146959	SQRTX2=	1.169470	SQRTX3=	1.181163
1/K=23.000	SQRTX1=	28.146783	SQRTX2=	1.178225	SQRTX3=	1.180058
1/K=23.500	SQRTX1=	28.146603	SQRTX2=	1.187107	SQRTX3=	1.179001

G= .01 FOR 1/K= 23.0925

REAL AND IMAGINARY ROOTS INTERSECT WHEN SQUARE ROOT OF X= 1.1799

W= 509.8030 V= 11772.6144

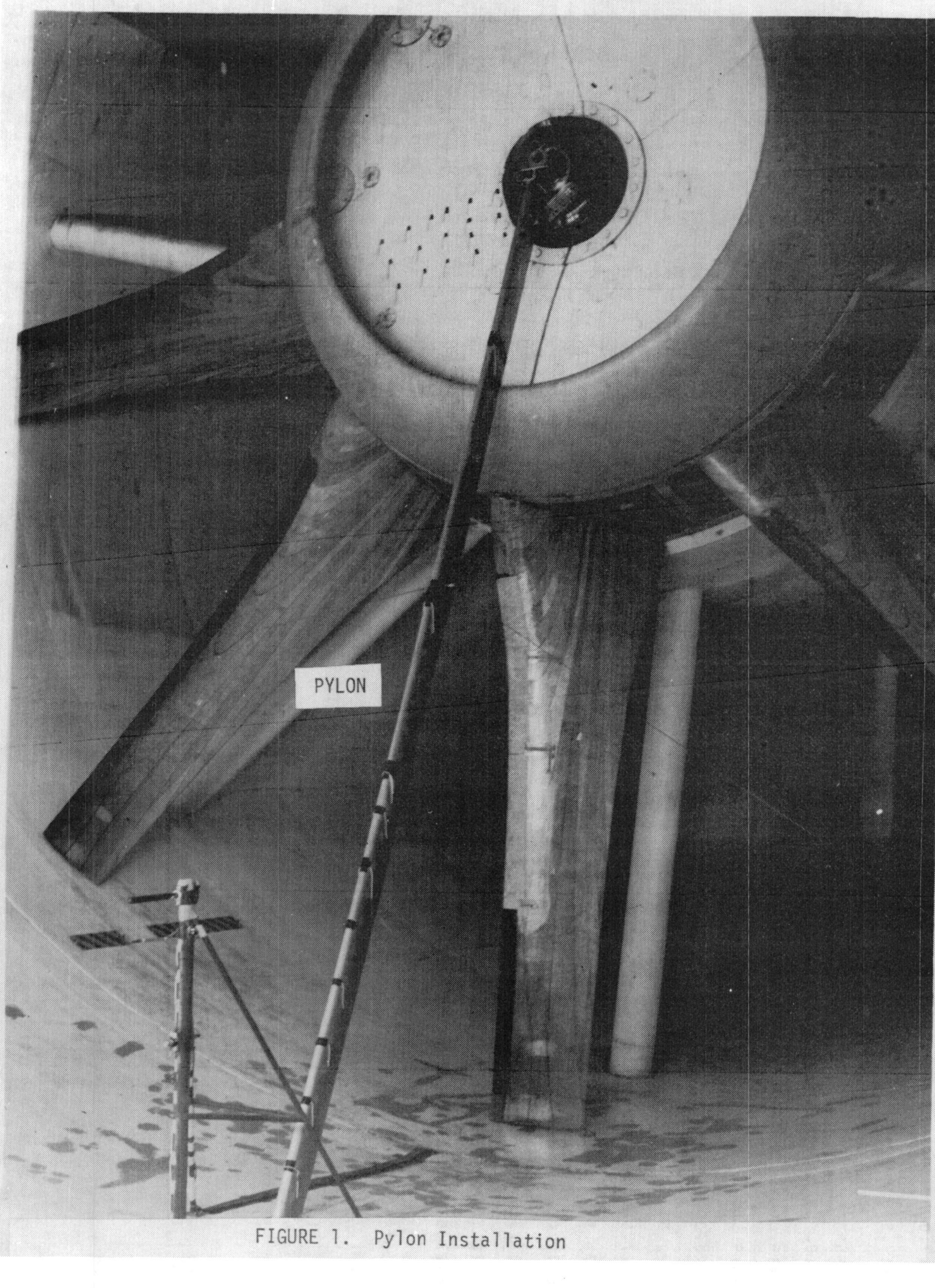


FIGURE 1. Pylon Installation

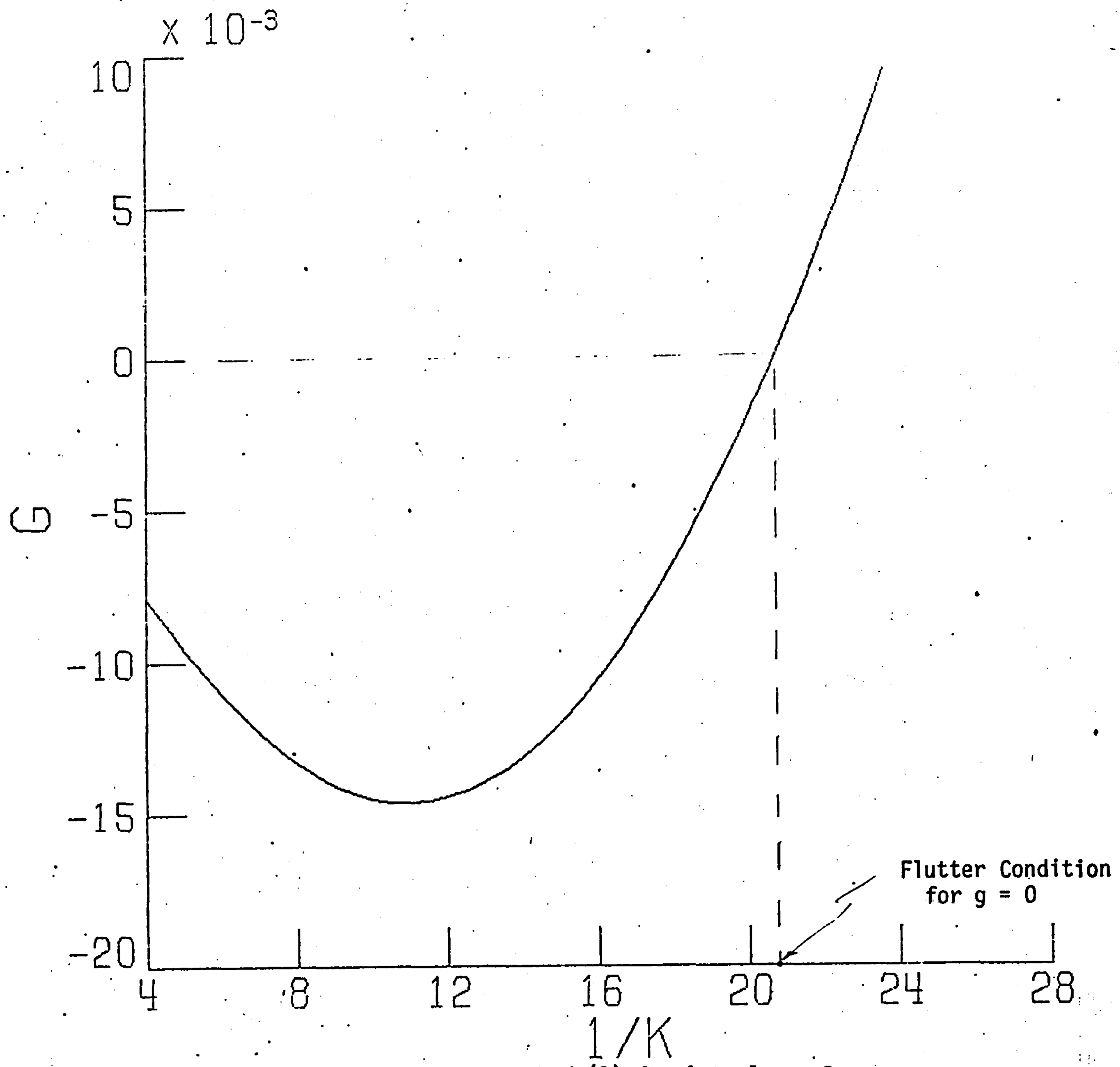


FIGURE 2. Variation of 1/K with g - Solution Method (1) for Actual g= 0.

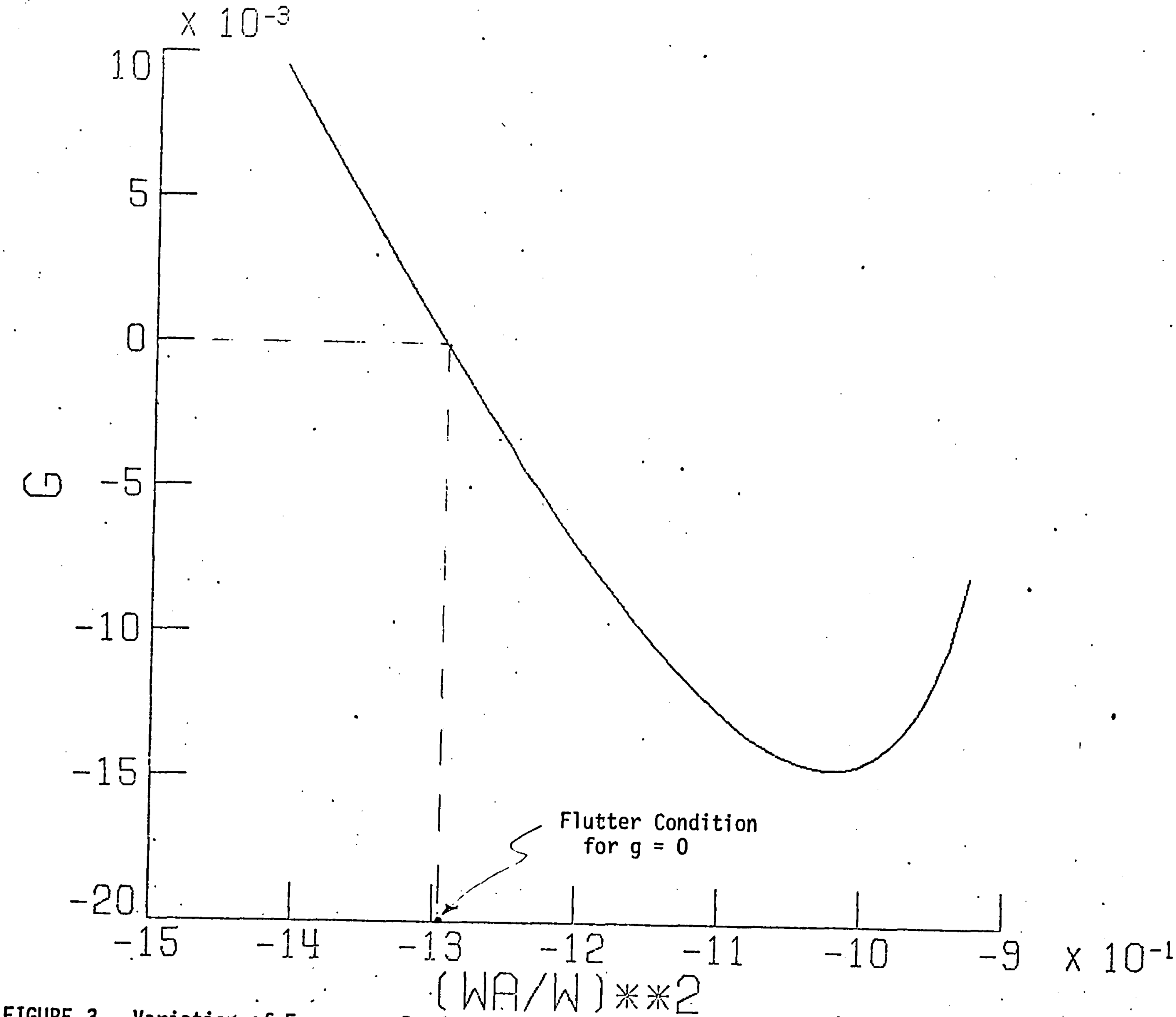


FIGURE 3. Variation of Frequency Ratio with g - Solution Method (1) for Actual g = 0.

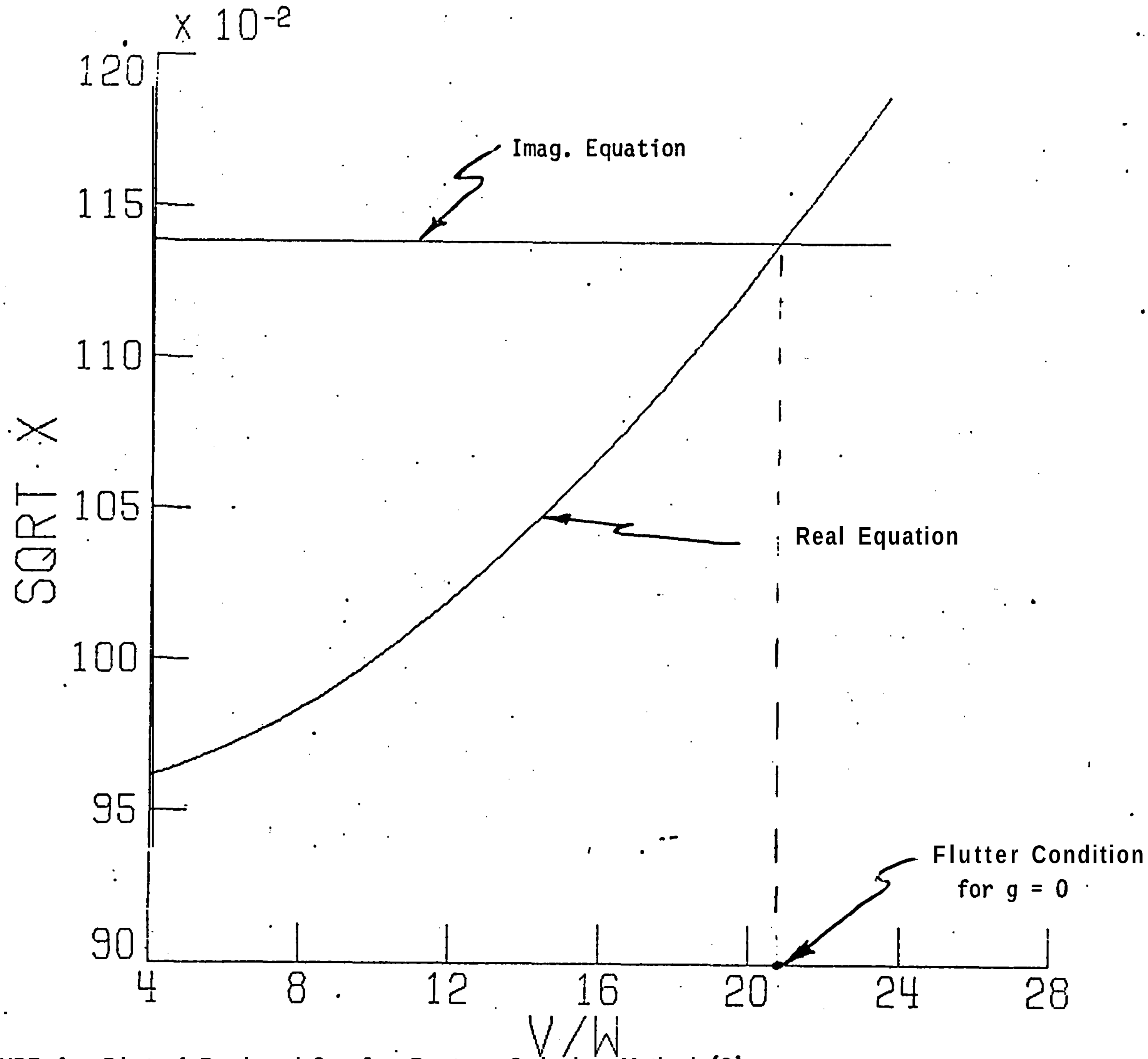


FIGURE 4. Plot of Real and Complex Roots - Solution Method (2).

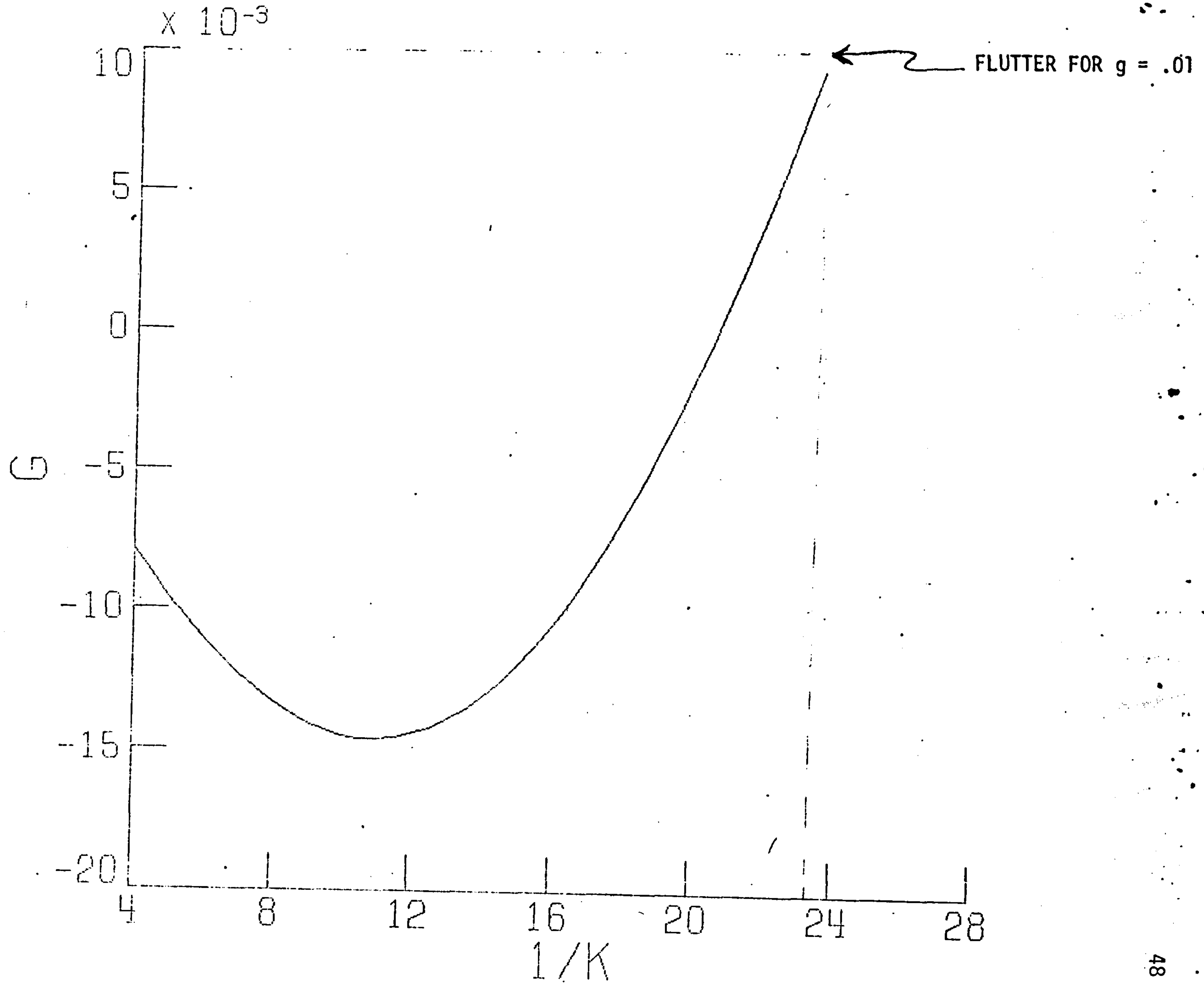


FIGURE 5. Variation of 1/K with g - Solution Method (1) for Actual g = .01.

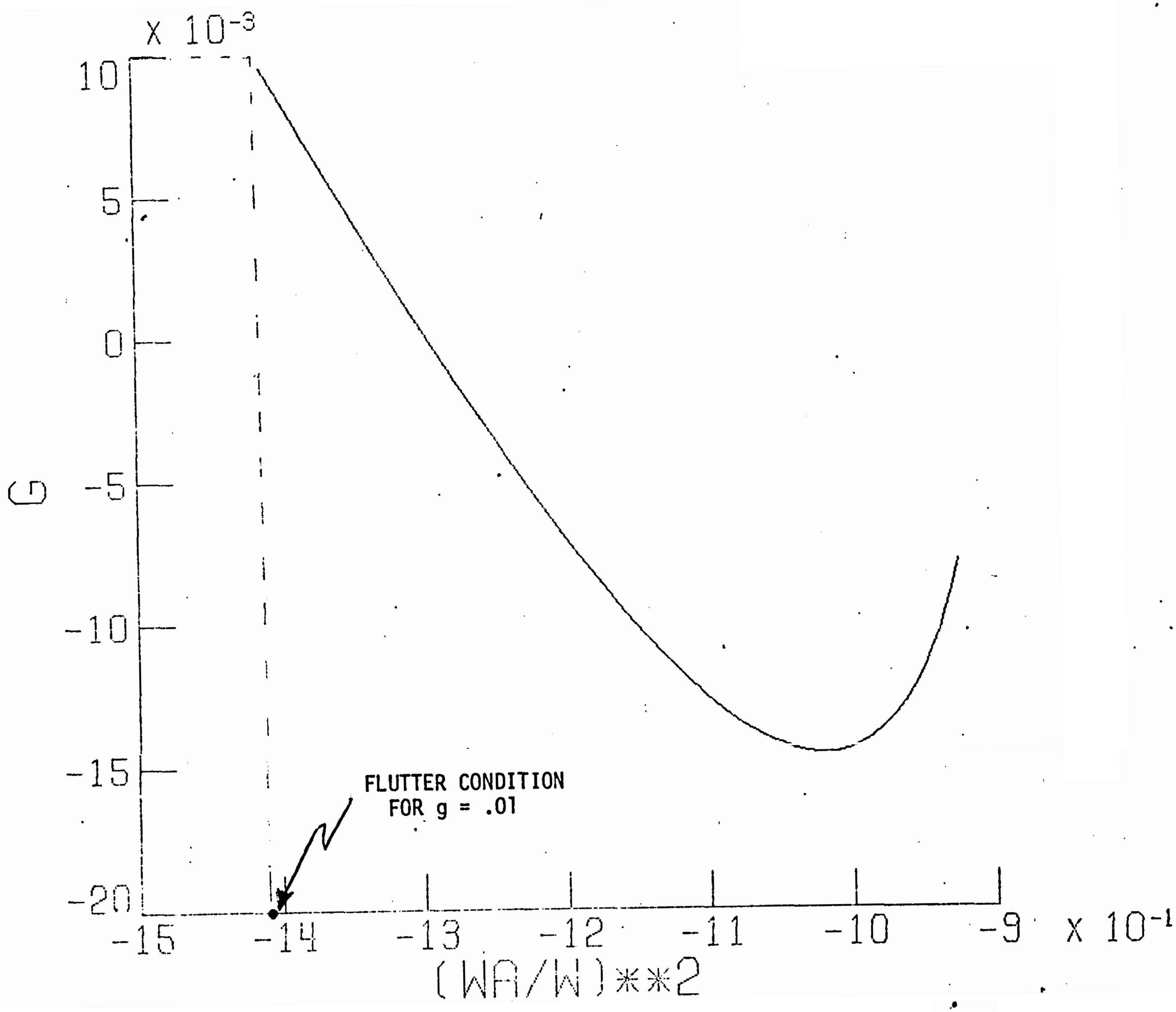


FIGURE 6. Variation of Frequency Ratio with g - Solution Method (1) for Actual g = .01.

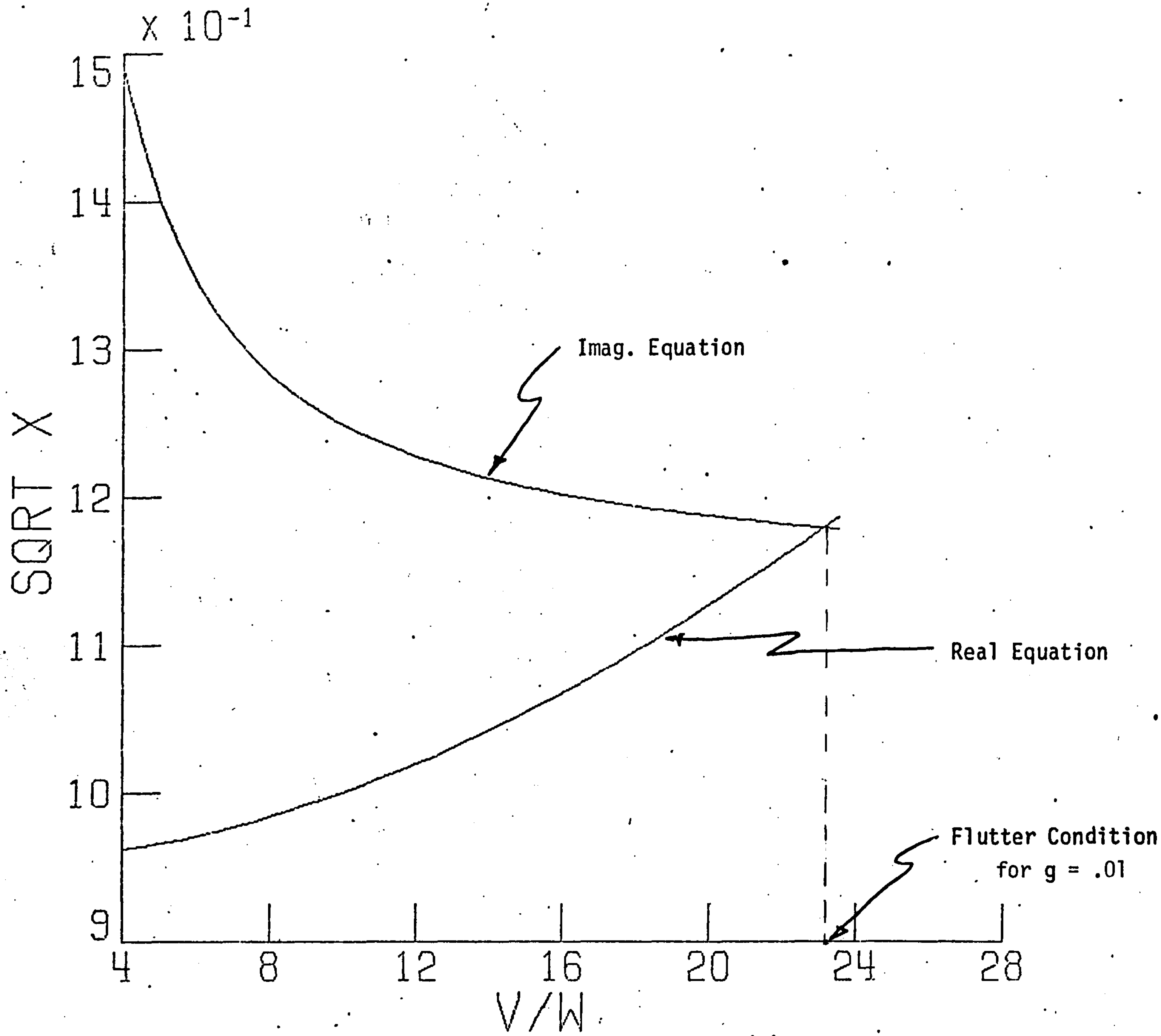


FIGURE 7. Plot of Real and Complex Roots - Solution Method (2).

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16. Abstract

The method given in this paper was developed to calculate flutter frequency and flutter speed for a problem with two degrees of freedom. Two different solutions for evaluating the flutter determinant are presented so that the results from each method could be compared. Although the method was developed for a particular problem application, it is sufficiently general to solve any flutter system that can be characterized by two degrees of freedom.

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